A strengthening of a theorem of Bourgain-Kontorovich II.

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Abstract

Zaremba's conjecture (1971) states that every positive integer number d can be represented as a denominator (continuant) of a finite continued fraction $\frac{b}{d} = [d_1, d_2, \dots, d_k]$, with all partial quotients $d_1, d_2, \dots, d_k$ being bounded by an absolute constant A. Recently (in 2011) several new theorems concerning this conjecture were proved by Bourgain and Kontorovich. The easiest of them states that the set of numbers satisfying Zaremba's conjecture with $A = 50$ has positive proportion in $\mathbb{N}$. In 2013 we proved this result with $A = 7$. In this paper the same theorem is proved with $A = 5$.
Keywords: continued fraction, continuant, exponential sums.

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