Renat Akhunzhanov, **On multidimensional Lagrange and Dirichlet spectrums.**

The one-dimensional Lagrange spectrum is the set

$$\mathbb{L} = \left\{ \lambda \in \mathbb{R} \mid \exists v \in \mathbb{R} : \liminf_{q \to \infty} q||qv|| = \lambda \right\}.$$

It is well known that the discrete part of the Lagrang spectrum has a sequence

$$\frac{1}{\sqrt{5}}, \frac{1}{2\sqrt{2}}, \frac{5}{\sqrt{221}}, \frac{13}{\sqrt{1517}}, \dots$$

This sequence converges to $\frac{1}{3}$. Also, the Lagrange spectrum has a segment $[0, \lambda^*]$, which calls Hall's ray (Hall first proved that $\lambda^* > 0$).

We consider the multi-dimensional Lagrange spectrum

$$\mathbb{L}_s = \left\{ \lambda \in \mathbb{R} \mid \exists \mathbf{v} \in \mathbb{R}^s : \liminf_{q \to \infty} q^{1/s} \max_{1 \leqslant i \leqslant s} ||qv_i|| = \lambda \right\}.$$

The one-dimensional Dirichlet spectrum $\mathbb{D}$ is defined as follows:

$$\mathbb{D} = \left\{ \lambda \in \mathbb{R} \mid \exists v \in \mathbb{R} : \limsup_{t \to \infty} t \cdot \min_{1 \leqslant q \leqslant t} ||qv|| = \lambda \right\}.$$

The structure of the one-dimensional Dirichlet spectrum was studied by many mathematicians. In particular $\mathbb{D} \subset [1/2+1/2\sqrt{5}, 1]$ and there exists so-called "discrete part of the spectrum". Moreover it is known that for a certain d^* one has $[d^*, 1] \subset \mathbb{D}$.

We consider s-dimensional Dirichlet spectrum with respect to Euclidean norm defined as

$$\mathbb{D}_s = \left\{ \lambda \in \mathbb{R} \mid \exists \mathbf{v} \in \mathbb{R}^s : \limsup_{t \to \infty} t \cdot \min_{1 \leqslant q \leqslant t} \left(\sum_{i=1}^{s} ||qv_i||^2 \right)^{s/2} = \lambda \right\}.$$

Multidimensional spectrum has rather different properties. In this talk we annonce the following results.

Theorem I(Akhunzhanov 2013). *For any* $t \leqslant \frac{1}{6 \cdot 4^s \cdot R \cdot C(s)(2s+3)}$ *the* s*-dimensional Lagrange spectrum has* $\lambda \in \mathbb{L}_s$ *where* $t \leq \lambda \leq t(1 + 16B \cdot t^{1+1/s})$.

Theorem II(Akhunzhanov, Shatskov 2013).

$$\mathbb{D}_2 = \left[0, \frac{2}{\sqrt{3}} \right].$$