

Martin Widmer, Asymptotic Diophantine approximation: The multiplicative case.

Let α and β be real irrational numbers and $0 < F < 1/30$. We discuss a precise estimate for the number of positive integers $q \leq Q$ that satisfy $\|q\alpha\| \|q\beta\| < F$. If we choose F as a function of Q we get asymptotics as Q gets large, provided FQ grows quickly enough in terms of the (multiplicative) Diophantine type of (α, β) , e.g., if (α, β) is a counterexample to Littlewood's conjecture then we only need that FQ tends to infinity. Our result yields a new upper bound on sums of reciprocals of products of fractional parts, and sheds some light on a recent question of L e and Vaaler.