International conference

DIOPHANTINE ANALYSIS

Astrakhan, Russia
July, 30 — August, 03 2012

PROGRAM and ABSTRACT BOOK

Program

Monday, July 30

08.00 – 09.15 Breakfast

09.25 – 09.30 Opening

Morning session – Chairman Carlo Viola

09.30 – 10.10 Yuri Nesterenko, “On some identities of Ramanujan type”

10.20 – 11.00 Michel Waldschmidt, “Families of Diophantine equations without non-trivial solutions”

Coffee break

11.20 – 12.00 Barak Weiss, “The Mordell-Gruber spectrum and homogeneous dynamics”

12.10 – 12.50 Jinpeng An, “Two dimensional badly approximable vectors and Schmidt’s game”

13.00 – 14.30 Lunch

Afternoon session – Chairman Barak Weiss

14.30 – 15.10 Dzmitry Badziahin, “The problems about badly approximable points and generalized Cantor sets”

15.20 – 16.00 Iekata Shiokawa, “Algebraic relations for the sixteen Eisenstein series”

16.10 – 16.40 Oleg German, “A point of view at transference principle”

Coffee break

17.00 – 17.20 Mariya Avdeeva, “On the average of partial quotients of continued fractions with a fixed denominator”

17.30 – 17.50 Victoria Zhuravleva, “Diophantine approximation with Fibonacci numbers”

18.00 – 18.30 Dmitrii Frolenkov, “Zaremba’s conjecture and Bourgain-Kontorovich’s theorem”

18.30 – 19.30 Dinner

Tuesday, July 31

08.00 – 09.30 Breakfast

Morning session – Chairman Yuri Nesterenko

09.30 – 10.10 Viktor Bykovskii, TBA

10.20 – 11.00 Carlo Viola, “Towards simultaneous approximations to $\zeta(2)$ and $\zeta(3)$”

Coffee break

11.20 – 12.00 Raffaele Marcovecchio, “The irrationality measure of $\zeta(2)$ revisited”

12.10 – 12.50 Henrietta Dickinson, “Algebraic points near smooth surfaces”

13.00 – 14.30 Lunch

Afternoon session – Chairman Michel Waldschmidt

14.30 – 15.10 Stephen Harrap, “Problems surrounding the mixed Littlewood conjecture for pseudo-absolute values”

15.20 – 16.00 Keijo Väänänen, “On arithmetic properties of q-series”

16.10 – 16.30 Pavel Kozlov, “About algebraic independence of functions of a certain class”

Coffee break

17.00 – 17.20 Alexandr Polyanskii, “On the irrationality measure of certain numbers”

17.30 – 18.00 Natalia Budarina, “Inhomogeneous Diophantine approximation on integer polynomials with non-monotonic error functions”

18.10 – 18.30 Faustin Adiceam, “From simultaneous Diophantine approximation on manifolds to Diophantine approximation under constraints”

18.30 – 19.30 Dinner

Wednesday, August 01

We have a boat excursion to Volga’s delt. The buses go from the University building at 08.00 Breakfast and lunch will be served during the excursion.

Thursday, August 02

08.00 – 09.30 Breakfast

Morning session – Chairman Iekata Shiokawa

09.30 – 10.10 Antanas Laurinčikas, “Universality of some functions related to zeta-functions of certain cusp forms”

10.20 – 11.00 Renata Macaitiene, “On the universality of zeta-functions of certain cusp forms”

Coffee break

11.30 – 12.10 Ilya Shkredov, “Additive structures in multiplicative subgroups”

12.20 – 13.00 Ryotaro Okazaki, “A refinement on a Theorem of Nagell on totally imaginary quartic Thue equations”

13.00 – 14.30 Lunch

Afternoon session – Chairman Antanas Laurinčikas

14.30 – 14.50 Andrey Illarionov, “Best approximations of linear forms and local minima of lattices”

15.00 – 15.20 Anton Shutov, “Multidimensional Hecke-Kesten problem”

15.30 – 16.10 Yitwah Cheung, “Khinchin-Levy constant for simultaneous approximation”

16.20 – 16.40 Mariya Monina, “On one arithmetic identity and it’s applications”

Coffee break

17.00 – 17.30 Simon Kristensen, “Metrical Musings on Littlewood and Friends”

17.40 – 18.20 Andrey Raigorodskii, “From combinatorial geometry to Ramsey theory”

18.30 Conference Party

Friday, August 03

08.00 – 09.30 Breakfast

Morning session – Chairman Henrietta Dickinson

09.30 – 09.50 Renat Akhunzhanov, “Two-dimensional Dirichlet spectrum”

10.00 – 10.20 Denis Shatskov, “On irrationality measure functions in average”

10.30 – 11.00 Dmitrii Gayfulin, “On the derivative of functions form Denjoy-Tichy-Uitz family”

Coffee break

11.30 – 12.00 Nikolay Moshchevitin, “Wolfgang Schmidt and his mathematics”

13.00 – 14.30 Lunch

List of participants

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Call for papers

We suppose to publish a volume of proceedings of our conference in 2013 in a special issue of

Moscow Journal of Combinatorics and Number Theory

Everybody is welcome to submit a paper.

The deadline for submission is December 31, 2012.
All the submissions will be refereed according to Journal's rules.

The website of the Journal is here:

http://mjcnt.phystech.edu/en/

ABSTRACTS of TALKS

Faustin Adiceam (Dublin, Ireland)

From simultaneous Diophantine approximation on manifolds to Diophantine approximation under constraints

Given a polynomial P in n variables with integer coefficients, the set of simultaneously τ – well approximable points lying on the manifold defined by P strongly depends on the rational solutions of the equation $P = 0$ as soon as τ is greater than the degree of P. Thus, when $n = 2$, Falting's Theorem relates Diophantine approximation on the curve defined by P to the genus of this curve. However, if the coefficients of P are no longer integers, none of these results remain true: the study of a specific example shows that we are led to deal with problems of Diophantine approximation with some specific – and seemingly new – constraints both on the numerators and the denominators of the rational approximants.

The results presented in this talk are the subject of ongoing work in the area.

Renat Akhunzhanov, Denis Shatskov (Astrakhan, Russia)

On two-dimensional Dirichlet spectrum[1]

One-dimensional Dirichlet spectrum $\mathbb{D}$ is defined as follows:

$$\mathbb{D} = \left\{ \lambda \in \mathbb{R} \mid \exists v \in \mathbb{R} : \limsup_{t\to\infty} t \cdot \min_{1\leqslant q\leqslant t} ||qv|| = \lambda \right\}.$$

The structure of one-dimensional Dirichlet spectrum was studied by many mathematicians. In particular $\mathbb{D} \subset [1/2 + 1/2\sqrt{5}, 1]$ and there exists so-called "discrete part of the spectrum". Moreover it is known that for a certain d^* one has $[d^*, 1] \subset \mathbb{D}$.

We consider s-dimensional Dirichlet spectrum with respect to Euclidean norm.

$$\mathbb{D}_s = \left\{ \lambda \in \mathbb{R} \mid \exists \mathbf{v} = (v_1, ..., v_s) \in \mathbb{R}^s : \limsup_{t\to\infty} t \cdot \min_{1\leqslant q\leqslant t} \left(\sum_{i=1}^{s} ||qv_i||^2 \right)^{s/2} = \lambda \right\}.$$

Multidimensional spectrum has rather different properties.

We prove that

$$\mathbb{D}_2 = \left[0, \frac{2}{\sqrt{3}} \right].$$

[1] Research is supported by the grant RFBR No. 12-01-00681-a.

Jinpeng An (Beijing, China)

Two dimensional badly approximable vectors and Schmidt's game

The notion of badly approximable vectors is a natural generalization of badly approximable numbers. In this talk, we will discuss winning properties of the set of two dimensional badly approximable vectors for Schmidt's game. As a consequence, we strengthen the recent breakthrough theorem of Badziahin-Pollington-Velani on countable intersections of such sets.

Mariia Avdeeva, Victor Bykovskii (Khabarovsk, Russia)

On the average of partial quotients of continued fractions with a fixed denominator[1]

Let $C(1), C(2), \ldots, C(n), \ldots$ be a sequence of real numbers (costs) such that $C \not\equiv 0$ and $|C(n)| \leqslant \log_2(n+1)$ for any $n \in \mathbb{N}$.

For rational $r \in (0,1]$ consider the continued fraction expansion

$$r = [0; q_1, q_2, \ldots, q_s] \qquad (q_i = q_i(r) \quad \text{are positive integers})$$

of length $s = s(r)$. We define

$$s_C(r) = \sum_{i=1}^{s(r)} C(q_i(r)) \qquad \text{and} \qquad M(C) = \sum_{n=1}^{\infty} C(n) \log\left(1 + \frac{1}{n(n+2)}\right).$$

Let $\mathcal{F}_N$ be a set of all irreducible fractions $r = m/n \in [0,1)$ with $n \leqslant N$ ($N \in \mathbb{N}$) and $\Phi(N)$ be a number of elements of the set $\mathcal{F}_N$.

It is known that exists (see [1])

$$\lim_{N\to\infty} \frac{1}{\Phi(N)} \sum_{r\in\mathcal{F}_N} \left| \frac{s_C(r) - \frac{12}{\pi^2} M(C) \log N}{\sqrt{\log N}} \right|^2 = D(C) > 0$$

and uniformly for $-\infty \leqslant \alpha < \beta \leqslant \infty$

$$\frac{1}{\Phi(N)} \# \left\{ r \in \mathcal{F}_N \;\middle|\; \alpha \leqslant \frac{s_C(r) - \frac{12}{\pi^2} M(C) \log N}{\sqrt{D(C) \log N}} \leqslant \beta \right\} =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\beta} e^{-\frac{x^2}{2}} dx + O\left(\frac{1}{\sqrt{\log N}}\right).$$

We use these results and ideas from [2] as base to prove the following estimation.

Theorem. *Put* $\mathbb{Z}'_d = \{a \in \mathbb{Z} \mid 1 \leqslant a \leqslant d,\ \gcd(a,d) = 1\}$ *and* $\varphi(d) = \#\mathbb{Z}'_d$. *Then* $\forall \varepsilon > 0$ *uniformly for* $\gamma \geqslant 1$

$$\frac{1}{\varphi(d)} \# \left\{ a \in \mathbb{Z}'_d \;\middle|\; \frac{\left| s_C(\frac{a}{d}) - \frac{12}{\pi^2} M(C) \log\, d \right|}{\sqrt{D(C) \log N}} \geqslant \gamma \right\} \ll_{C,\varepsilon} \gamma^{-\frac{1}{4}} e^{-\frac{\gamma^2}{2}} + \log^{-\frac{1}{2}+\varepsilon} d.$$

[1]Supported by the RFBR (project No. 11-01-12004-ofi-m-2011), by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-1922.2012.1).

If $C(n) \equiv 1$ and $1 \leqslant \gamma \leqslant \log^{\frac{1}{2}-\varepsilon} d$ then we get a strengthened result implied by Chebyshev's inequality and the estimation

$$\frac{1}{d} \cdot \sum_{a=1}^{d-1} \left| s\left(\frac{a}{d}\right) - \frac{12}{\pi^2} \log\, 2 \cdot \log\, d \right|^2 \ll \log\, d$$

which was proved in [2].

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Dzmitry Badziahin (York, UK)

The problems about badly approximable points and generalized Cantor sets

We firstly introduce Cantor-type sets which are constructed similarly to the middle-third Cantor set but in much more general way. It appears that they satisfy some very nice properties. Firstly one can estimate their Hausdorff dimension (especially its lower bound). And the intersection of two Cantor-type sets can often be considered as another Cantor-type set which helps to estimate the "size" of their intersection.

Secondly we'll show that various sets of badly approximable points can be described in terms of Cantor-type sets which allow us to achieve highly nontrivial results about their structure. For example we consider the sets of badly approximable points on the plane:

$$\mathbf{Bad}(i,j) := \{(\alpha,\beta) \in \mathbb{R}^2 \mid \liminf_{q\to\infty} q \cdot \max\{||q\alpha||^{1/i}, ||q\beta||^{1/j}\} > 0\}$$

where $i, j \geqslant 0, i + j = 1$.

In particular the explained method enabled D.B., Pollington and Velani in 2010 to prove famous Schmidt problem which states that the intersection $\mathbf{Bad}(i_1, j_1) \cap \mathbf{Bad}(i_2, j_2)$ is always non-empty. Recently it also allowed D.B. and Velani to show that the set $\mathbf{Bad}(i, j) \cap \mathcal{C}$ of badly approximable points on any non-degenerate curve $\mathcal{C}$ is of full Hausdorff dimension which answers positively to the question of Davenport posed in 1964. Some other applications of Cantor-type sets will be mentioned as well.

Natalia Budarina (Dublin, Ireland)

Inhomogeneous Diophantine approximation on integer polynomials with non–monotonic error function

We consider the problem of approximating real numbers by polynomials with a non–monotonic error function. First some notation is needed. Throughout, $P \in \mathbb{Z}[f]$ with

$$P(f) = a_n f^n + \cdots + a_1 f + a_0$$

will be an integer polynomial with degree $\deg P = n$ and height $H(P) = \max\limits_{0 \leqslant j \leqslant n} |a_j|$. Let

$$P_n = \{P \in \mathbb{Z}[f] : \deg P \leqslant n\}.$$

Let $\mu(A)$ denote the Lebesgue measure of a measurable set $A \subset \mathbb{R}$.

Throughout d will be a fixed real number. Define a real valued function $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ and let $\mathcal{L}_{n,d}(\Psi)$ be the set of $x \in \mathbb{R}$ such that the inequality

$$|P(x) + d| < \Psi(H(P)) \tag{1}$$

has infinitely many solutions $P \in \mathcal{P}_n$. The set $L_{n,d}(\Psi)$ corresponds to the inhomogeneous setting. In turn, with $d = 0$ the corresponding set reduces to the homogeneous setting and is denoted by $\mathcal{L}_n(\Psi)$.

The main result is the following statement.

Theorem 1. *For $n \geqslant 3$*

$$\mu(\mathcal{L}_{n,d}(\Psi)) = 0$$

if the sum $\sum\limits_{h=1}^{\infty} h^{n-1}\Psi(h)$ converges.

There are many results regarding this problem when Ψ is monotonic and $d = 0$. For $\Psi(H) = H^{-w}$, $w > n$, and $d = 0$ the theorem was proved by Sprindžuk [14]. For a general monotonic function Ψ such that $\sum\limits_{h=1}^{\infty} \Psi^{1/n}(h) < \infty$ and $d = 0$ it was proved by A. Baker [2] who further conjectured that $\mu(L_n(\Psi)) = 0$ if the sum $\sum\limits_{h=1}^{\infty} h^{n-1}\Psi(h)$ converges. This was then proved in 1989 by Bernik [8]; later, Beresnevich [3] proved the corresponding divergence result. The first time inequality (1) for any $d \in \mathbb{R}$ was considered was in [9] and a similar question in the p-adic case was answered in [10].

The above problems can be considered as problems concerning Diophantine approximation on the Veronese curve $\mathcal{V}_n = \{(x, x^2, \ldots, x^n) : x \in \mathbb{R}\}$. In 1998 Kleinbock and Margulis [13] established the Baker – Sprindzuk conjecture concerning homogeneous Diophantine approximation on manifolds. An inhomogeneous version of the theorem of Kleinbock and Margulis was then proved by Beresnevich and

Velani [7]. The significantly stronger Groshev type theory for dual Diophantine approximations on manifolds has been established in [4], [5], [11] for the homogeneous case and in [1] for the inhomogeneous case. In all of these results the function Ψ was assumed to be monotonic. In 2005 Beresnevich [6] showed that Theorem 1 holds without the condition that Ψ is monotonic for $d = 0$; he conjectured that the result should also hold for any non – degenerate curve in Euclidean space. This was proved in [12]. We extend this last result to the inhomogeneous setting for the Veronese curve $\mathcal{V}_n$. Note that using result from [12] (by taking $\mathbf{f} = (1, x, x^2, \ldots, x^n, d)$ and $\mathbf{a} = (a_0, a_1, \ldots, a_n, 1)$) we can obtain that $\mu(\mathcal{L}_{n,d}(\Psi)) = 0$ if $\sum\limits_{h=1}^{\infty} h^n \Psi(h) < \infty$. But the condition $\sum\limits_{h=1}^{\infty} h^{n-1} \Psi(h) < \infty$ in the Theorem 1 gives less restrictions.

References

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Viktor Bykovskii, Mariya Monina (Khabarovsk, Russia)

On the one arithmetic identity and it's applications[1]

In the series of eighteen papers published in “Journal de mathématiques pures et appliquées” in 1858-1865 and joined by the common name “Sur quelques formules générales qui peuventétre utiles dans la théorie des nombres”, the French mathematician Joseph Liouville gave without proof a large number of arithmetic identities (their list can be found in [1]). Using it, in a large quantity of short notes, published the same years and as without proofs, he calculated the number of representations of a positive number by special quadratic forms. Liouville's methods were recreated in papers of Baskakov, Nazimov, Uspensky and other authors and got further development (see [2], [3] and [4]).

In this direction of research we formulate a new result. Adding it by ideas from [5], it is possible to obtain the new elementary arithmetic proofs of some classical identities from the theory of elliptic and theta-functions.

Let $x = (x_1, \ldots, x_s)$ be a system of s independent variables, $\Omega \subset \mathbb{R}^s$, $Q(x)$ be a quadratic form and $L(x)$ be a nondegenerate linear form with the integer coefficients. Assume that J, U_- and U_+ are integer linear automorphisms of Q (it means that form Q is invariant under substitutions $x \to J(x)$ and $x \to U_\pm(x)$), for which J, U_- and U_+ are bijections on Ω, $\left\{u \in \Omega \;\middle|\; L(u) < 0\right\}$ and $\left\{u \in \Omega \;\middle|\; L(u) > 0\right\}$, $L(J(x)) = -L(x)$, $\quad U_-(J(x)) = J(U_+(x))$.

Theorem. *Let $\varPhi : \mathbb{Z}^s \to A$ be a function with values in additive abelian group A, which is distinct from zero only for certain finite number of values $m \in \mathbb{Z}^s$, and $\varPhi \circ R = -\varPhi$ for automorphism $R = U_- \circ J = J \circ U_+$. Then for any integer l*

$$\sum_{\substack{m\in\mathbb{Z}^s\cap\Omega\\ Q(m)=l}} \varPhi(m) = \sum_{\substack{m\in\mathbb{Z}^s\cap\Omega\\ Q(m)=l;\ L(m)=0}} \varPhi(m).$$

Specializing $\varPhi$, Ω, L, J, U_-, U_+ for the form $Q(x_1, x_2, x_3) = x_2^2 + x_1x_3$, we obtain two identities

$$\sum_{\substack{b^2+ac=d\\ a \text{ is even}}} af(b) + 4 \sum_{\substack{b^2+ac=d\\ a \text{ is odd}}} (-1)^c af(c-b) = \begin{cases} 0, & \text{if } d \neq n^2;\\ -2n^2 f(n), & \text{if } d = n^2, \end{cases}$$

where $f(b) = x^b + x^{-b}$, $a, c, d, n \in \mathbb{N}$, $b \in \mathbb{Z}$, and

$$-\frac{1}{4} \sum_{\substack{b^2+ac=d\\ a\equiv 0 (\mathrm{mod}\ 3)}} a\,\chi_{-3}(b)\left(\left(1+(-1)^a\right)g(b) + 2\left(2+(-1)^{a+c}+(-1)^c\right)g(b-c)\right) =$$

$$= \begin{cases} \chi_{-3}(n)(n^2-1)g(n), & \text{if } d = n^2;\\ 0, & \text{if } d \neq n^2, \end{cases}$$

[1]Supported by the Grants Council (under RF President) for State Aid of Leading Scientific Schools (grant NSh-1922.2012.1).

where $g(b) = x^b - x^{-b}$ and χ_{-3} is quadratic character modulo 3.

These identities are equivalent to logarithmic derivatives with respect to q of classical identities of Jacobi – Gauss

$$\prod_{k=1}^{\infty} \left(1-q^{2k}\right)\left(1+xq^{2k-1}\right)\left(1+x^{-1}q^{2k-1}\right) = \sum_{b=-\infty}^{\infty} x^b q^{b^2}.$$

and of Watson – Gordon

$$q(x-x^{-1})\prod_{k=1}^{\infty} \left(1-q^{6k}\right)\left(1-xq^{3k}\right)\left(1-x^{-1}q^{3k}\right)\left(1+xq^{6k}\right)\left(1+x^{-1}q^{6k}\right) =$$

$$= \sum_{b=-\infty}^{\infty} \chi_{-3}(b) x^b q^{b^2}.$$

References

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Yitwah Cheung (San Francisco, USA)

Khinchin-Levy constant for simultaneous approximation

Khinchin showed there is a constant L such that for almost every real number the limit $\frac{\log q_k}{k}$ exists and is equal to L. Here, q_k is the sequence of best approximation denominators. This constant was computed by Levy to be $\frac{\pi^2}{12 \log 2}$. In this talk, I will describe a Poincare section of the flow on SL(3,R)/SL(3,Z) induced by a certain diagonal flow and use it to prove the existence of Khinchin-Levy constant for pairs of real numbers with respect to a generic inner product norm.

This is joint work with Nicolas Chevallier.

Henrietta Dickinson (Dublin, Ireland)
Algebraic points near smooth surfaces

This is joint work with Vasili Bernik.

In this talk the question of counting algebraic points near smooth surfaces will be addressed. A lower bound for the number of such points within a certain distance of the surface will be found using some recent measure theoretical results from metric Diophantine approximation. In particular it will be shown that the set of points in a small rectangle T, which satisfy some polynomial conditions on T has relatively large measure ($> 3|T|/4$).

The main part of the proof is to show that the set of points which have small value and small derivative for all polynomials of given degree and height has small measure.

Dmitrii Frolenkov, Igor Kan (Moscow, Russia)

On Zaremba conjecture and Bourgain & Kontorovich theorem

In 1971 Zaremba conjectured that every integer number can be represented as a denominator of a finite continued fraction with partial quotients being bounded by an absolute constant A ($A = 5$). In the recent paper by Bourgain & Kontorovich several breakthrough results concerning Zaremba conjecture have been proved. The easiest of them states that the set of numbers satisfying Zaremba conjecture with $A = 50$ has positive density in $\mathbb{N}$. The proof ot this theorem is based on the estimates of exponential sums and on the spectral theory. We have reduced the constant A to $A = 7$ and also proved a result with $A = 13$ which does not refer to the spectral theory.

Oleg German (Moscow, Russia)

A point of view at transference principle[1]

In 1937 Kurt Mahler proved the following famous theorem, which implies many classical transference results.

Theorem 1. *Let $f_1(\vec{z}),\dots,f_d(\vec{z})$ be d linearly independent homogeneous linear forms in $\vec{z}\in\mathcal{R}^d$ and let $g_1(\vec{w}),\dots,g_d(\vec{w})$ be d linearly independent homogeneous linear forms in $\vec{w}\in\mathcal{R}^d$ of determinant D. Suppose that all the coefficients of the bilinear form*

$$\Phi(\vec{z},\vec{w})=\sum_{i=1}^{d} f_i(\vec{z})g_i(\vec{w})$$

are integers. Then, if there is a non-zero $\vec{z}\in\mathbb{Z}^d$ such that

$$|f_1(\vec{z})|=\lambda_1,\qquad |f_i(\vec{z})|\le\lambda_i,\qquad i=2,\dots,d, \tag{1}$$

then there is a non-zero $\vec{w}\in\mathbb{Z}^d$ such that

$$|g_1(\vec{z})|\le(d-1)\lambda/\lambda_1,\qquad |g_i(\vec{z})|\le\lambda/\lambda_i,\qquad i=2,\dots,d, \tag{2}$$

where

$$\lambda=\left(|D|\prod_{i=1}^{d}\lambda_i\right)^{\frac{1}{d-1}}.$$

At the talk we shall propose a point of view at the transference phenomenon from which Theorem 1 becomes a boundary point of a family of results. Particularly, the factor $d-1$ in (2) can be shifted to any of the inequalities in (2) without changing (1). This in its turn can give us some additional information concerning the higher successive minima of the body defined by (2).

[1] This research was supported by RFBR grant No. 12–01–31106 and by the grant of the President of Russian Federation No. MK–5016.2012.1

Dmitrii Gayfulin (Moscow, Russia)

On the derivative of functions form Denjoy-Tichy-Uitz family

The Denjoy-Tichy-Uitz family of functions $\{g_\lambda(x)\}$ is a generalization of the famous Minkowski question-mark function $?(x)$. Each function of the family is a strictly increasing function, it is continuous and maps the segment $[0,1]$ onto itself. For $\lambda \in (0,1)$ the function $g_\lambda(x)$ is defined as follows. First of all

$$g_\lambda\left(\frac{0}{1}\right) = 0, \ g_\lambda\left(\frac{1}{1}\right) = 1,$$

for aly $\lambda \in [0,1]$. Then if $g_\lambda(x)$ is defined for two consecutive Farey fractions $\frac{p}{q} < \frac{r}{s}$ we put

$$g_\lambda\left(\frac{p+q}{r+s}\right) = (1-\lambda)g_\lambda\left(\frac{p}{q}\right) + \lambda g_\lambda\left(\frac{r}{s}\right).$$

For irrational $x \in [0,1]$ the function $g_\lambda(x)$ is defined by continuous argument. One can easily see that for $\lambda = \frac{1}{2}$ the function $g_\lambda(x)$ coincides with $?(x)$. It is a well-known fact that for all λ the derivative of $g'_\lambda(x)$ can take only two values 0 and $+\infty$. It's interesting to find two conditions for x such that if the first condition is satisfied then the derivative of $g'_\lambda(x)$ exists and equals 0, and if the second condition is satisfied then the derivative of $g'_\lambda(x)$ exists and equals $+\infty$. I am going to introduce some results about these conditions for the values $\lambda = \frac{\sqrt{5}-1}{2}$ and $\lambda = \frac{3-\sqrt{5}}{2}$.

Stephen Harrap (Aarhus, Denmark)

Problems surrounding the mixed Littlewood conjecture for pseudo-absolute values

I will discuss a variant of the mixed Littlewood Conjecture, namely one relating to a “pseudo-absolute value” sequence, and how it can be solved under certain restrictions on this sequence. The method appeals to a measure rigidity theorem of Lindenstrauss and relies upon being able to prove the existence of some ergodic measure with positive entropy. Time permitting, I will also describe a metrical result complementing the mixed Littlewood Conjecture itself.

Andrei Illarionov (Khabarovsk, Russia)
Best approximations to linear forms and local minima of lattices[1]

Let $f : \mathbb{R}^n \to [0, +\infty)$ be a radial, continuous, piecewise differentiable function.

A nonzero vector $(u, v) = (u_1, \ldots, u_n, v) \in \mathbb{Z}^n \times \mathbb{Z}$ is called a f – best approximation of the linear form $L_\alpha(x) = \alpha_1 x_1 + \ldots + \alpha_n x_n$ if there is no nonzero vector $(u', v') \in \mathbb{Z}^n \times \mathbb{Z}$ such that

$$|L(u') - v'| \leqslant |L(u) - v|, \quad f(u') \leqslant f(u),$$

and at least of the inequalities is strict.

Let $\mathfrak{B}_f(\alpha)$ be the set of f – best approximations (u, v) of the linear form L_α, and

$$\mathfrak{B}_f(\alpha, P) = \{(u, v) \in \mathfrak{B}_f(\alpha) : f(u) \leqslant P\} \quad \text{for } P \in (1, +\infty).$$

The set $\mathfrak{B}_f(\alpha, P)$ is finite, and $\#\mathfrak{B}_f(\alpha, P) = O_f(\ln P + 1)$. Here $\#X$ is the number of elements in a set X.

Take any real $R > 1$. Let

$$\Delta_n(R) = \left\{ \left(\frac{P_1}{Q}, \ldots, \frac{P_n}{Q} \right) : \quad 0 \leqslant P_i < Q \leqslant R, \quad i = \overline{1, n} \right\},$$

where P_i, Q are integer. We define the following average values

$$E_f(R, P) = \frac{1}{\#\Delta_n(R)} \sum_{\alpha \in \Delta_n(R)} \#\mathfrak{B}_f(\alpha, P),$$

$$E_f(R) = \frac{1}{\#\Delta_n(R)} \sum_{\alpha \in \Delta_n(R)} \#\mathfrak{B}_f(\alpha),$$

$$\mathcal{E}_f(P) = \int_{[0,1)^s} \#\mathfrak{B}_f(\alpha, P)\, d\alpha.$$

From famous properties of continuous fractions it follows that

$$E_f(R) = C_1 \ln R + O(1), \quad \mathcal{E}_f(P) = C_1 \ln P + O(1), \quad C_1 = \frac{24 \cdot \ln 2}{\pi^2},$$

if $n = 1$, $f(x) = |x|$ (one-dimensional case).

We prove the multidimensional generalization of these results.

[1]This work was supported by Far East Division of Russian Academy of Sciences (projects No. 12-III-B-01M-004) and the Grants Counsil (under RF President) for State Aid of Leading Scientific Schools (grant NSh-1922.2012.1).

Theorem. *Let $f : \mathbb{R}^n \to [0, +\infty)$ be a radial, continuous, piecewise differentiable function ($n \geqslant 2$). Then*

$$E_f(R, P) = \mathcal{C}_f \cdot \ln P + O_f(1) \qquad \textit{for } 1 < P \ll R^{1/n}, \tag{3}$$

$$\mathcal{E}_f(P) = \mathcal{C}_f \cdot \ln P + O_f(1) \qquad \textit{for } 1 < P. \tag{4}$$

Here $\mathcal{C}_f$ is the positive constant depending only on the f. The formula for $\mathcal{C}_f$ is omit because of its complexity.

From (3) it follows that

$$E_f(R) = \frac{\mathcal{C}_f}{n} \cdot \ln R + O_f(1) \quad \text{for any } R > 1.$$

The proof of (3) is based on the design of local cylindrical minima of lattices (see A. A. Illarionov, "On cylindrical minima of integer lattices", *Algebra i Analiz,* **24**:2 (2012) [in Russian]). Calculation of the sum $\sum\limits_{\alpha \in \Delta_n(R)} \#\mathfrak{B}_f(\alpha, P)$ we reduce to the determination of the number of integer matrices $M = \big(\!\big(m_{ij}\big)\!\big) \in \Omega_f$ such that

$$|\det M| \in [1, R], \quad \text{g.c.d.}(M_1, \ldots, M_{n+1}) = 1, \quad f(m_{11}, \ldots, m_{n1}) \leqslant P,$$

where Ω_f is a subset of $\mathrm{GL}_{n+1}(\mathbb{R})$, M_j is the $(n+1, j)$ – cofactor of matrix M.

The formula (4) is obtained by passing to the limit to (3) at $R \to \infty$.

Pavel Kozlov (Moscow, Russia)

About algebraic independence of functions of a certain class

In this work I describe the classical Ramanujan's functions $P(z)$, $Q(z)$, $R(z)$ and introduce a family of functions which have very similar properties. I prove algebraic independence of functions of this family. Finally, I deduce a boundary on multiplicity of zeros for polynomials of these functions.

Simon Kristensen (Aarhus, Denmark)

Metrical Musings on Littlewood and Friends

The celebrated Littlewood conjecture in Diophantine approximation concerns the simultaneous approximation of two real numbers by rationals with the same denominator. A cousin of this conjecture is the mixed Littlewood conjecture of de Mathan and Teulie, which is concerned with the approximation of a single real number, but where some denominators are preferred to others. Other related problems include an inhomogeneous version of the Littlewood conjecture, and a variety of hybrids sit in between these problems.

In the talk, we will derive a metrical result extending work of Pollington and Velani on the Littlewood conjecture. Our result implies the existence of an abundance of numbers satisfying many such conjectures.

This is joint work with Alan Haynes and Jonas Lindstrom Jensen.

Antanas Laurinčikas (Vilnius, Lithuania)

Universality of some functions related to zeta-functions of certain cusp forms

Let $F(z)$ be a normalized Hecke-eigen cusp form of weight κ for the full modular group with Fourier series expansion

$$F(z) = \sum_{m=1}^{\infty} c(m) \mathrm{e}^{2\pi i m z}, \quad c(1) = 1.$$

The associated zeta-function $\zeta(s, F)$ is defined, for $\sigma > \frac{\kappa+1}{2}$, by

$$\zeta(s, F) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s},$$

and is analytically continued to an entire function. In [1], it has been shown that $\zeta(s, F)$ is universal in the Voronin sense, i.e., its shifts $\zeta(s + i\tau, F)$, $\tau \in \mathbb{R}$, approximate uniformly on compact subsets of the strip $D = \left\{s \in \mathbb{C} : \frac{\kappa}{2} < \sigma < \frac{\kappa+1}{2}\right\}$ any non-vanishing analytic function.

In the report, we consider a set of operators Φ, as large as possible, for which the function $\Phi(\zeta(s, F))$ satisfies the universality property. This is a joint work with K. Matsumoto and J. Steuding.

We give one example. Denote by $H(D)$ the space of analytic functions on D with the topology of uniform convergence on compacta.
For $a_1, \ldots, a_r \in \mathbb{C}$, let $H_{a_1,\ldots,a_r;\Phi(0)}(D) = \{g \in H(D) : \ g(s) = a_j, \ j = 1, \ldots, r\} \cup \{\Phi(0)\}$ and $S_F = \{g \in H(D) : \ g(s) \neq 0 \text{ or } g(s) \equiv 0\}$.

Theorem 1. *Suppose that $\Phi : \ H(D) \to H(D)$ is a continuous operator such that $\Phi(S_F) \supset H_{a_1,\ldots,a_r;\Phi(0)}(D)$. When $r = 1$, let $K \subset D$ be a compact set with connected complement, and let $f(s)$ be a continuous and $\neq a_1$ function on K which is analytic in the interior of K. If $r \geqslant 2$, let $K \subset D$ be an arbitrary compact subset, and $f \in H_{a_1,\ldots,a_r;\Phi(0)}(D)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{\tau \in [0, T] : \ \sup_{s\in K} |\Phi(\zeta(s + i\tau, F)) - f(s)| < \varepsilon\right\} > 0.$$

Other classes of operators Φ are given in [2].

References

[1] A. Laurinčikas and K. Matsumoto, The universality of zeta – functions attached to certain cusp forms, *Acta Arith.* **98** (2001), 345 – 359.

[2] A. Laurinčikas, K. Matsumoto and J. Steuding, Universality of some functions related to zeta-functions of certain cusp forms, *Osaka Math. J.* (to appear).

Renata Macaitiene (Šiauliai, Lithuania)
On the universality of zeta-functions of certain cusp forms

One remarkable property of zeta-functions is that every analytic function can by approximated uniformly on compact sets by shifts of these functions. By the Mergelyan theorem, every continuous function on a compact subset $K \subset \mathbb{C}$ which is analytic in the interior of K can be approximated with a given accuracy uniformly on K by polynomials. Moreover, it is known that there exist functions whose shifts approximate on compact subsets of some region any analytic functions. First this was observed by S. M. Voronin who proved the above approximation property for the Riemann zeta-function.

In the talk, we will discuss about the simultaneous approximation of a collection of analytic functions by shifts of zeta-functions attached to cusp forms with respect to the Hecke subgroup with Dirichlet character. More precisely, let $SL_2(\mathbb{Z})$, as usual, denote the full modular group, κ and q be two positive integers, $\Gamma_0(q) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) : c \equiv 0 (\operatorname{mod} q) \right\}$ be the Hecke subgroup, and let χ be a Dirichlet character modulo q. We consider a cusp form $F(z)$ of weight κ with respect to $\Gamma_0(q)$ with character χ. This means that $F(z)$ is a holomorphic function in the upper half-plane $\Im z > 0$, for all $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(q)$ satisfies some functional equation and at infinity has the Fourier series expansion $F(z) = \sum\limits_{m=1}^{\infty} c(m) \mathrm{e}^{2\pi i m z}$. We attach to the form $F(z)$ the zeta-function

$$\zeta(s, F) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s}.$$

By the well-known Hecke results, the latter Dirichlet series converges absolutely for $\sigma > \frac{\kappa+1}{2}$, and is analytically continued to an entire function. Moreover, the function $\zeta(s, F)$ satisfies the functional equation and we have that $\left\{ s \in \mathbb{C} : \frac{\kappa-1}{2} \leqslant \sigma \leqslant \frac{\kappa+1}{2} \right\}$ is the critical strip for the function $\zeta(s, F)$. Additionally, we assume that the function $F(z)$ is primitive, thus is a simultaneous eigenfunction of all Hecke operators and $c(m)$ is the corresponding eigenvalues.

Let meas $\{A\}$ denote the Lebesgue measure of a measurable set $A \subset \mathbb{R}$ and
$D_\kappa = \left\{ s \in \mathbb{C} : \frac{\kappa}{2} < \sigma < \frac{\kappa+1}{2} \right\}$. Then, we have the following result.

Theorem. *Let $K \subset D_\kappa$ be a compact subset with connected complement, and let $f(s)$ be a continuous non-vanishing function on K which is analytic in the interior of K. Then, for every $\varepsilon > 0$,*

$$\liminf_{T \to \infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |\zeta(s + i\tau, F) - f(s)| < \varepsilon \right\} > 0.$$

Raffaele Marcovecchio (Pisa, Italy)

The irrationality measure of $\zeta(2)$ revisited

A family of double integrals over $(0,1)^2$ belonging to $\mathbb{Q} + \mathbb{Z}\zeta(2)$, whose rational parts have controlled denominators, yields the best known irrationality measure of $\zeta(2)$, namely 5.441242... (Rhin-Viola, 1996). Its arithmetical structure can now be obtained through repeated partial integrations performed with respect to any variable. It is also proved that the permutation group acting on this family of integrals and isomorphic to $\mathfrak{S}_5$ is generated by four involutions (two for each variable) acting on certain Legendre-type polynomials, who naturally appear in the partial integration method.

Nikolay Moshchevitin (Moscow, Russia)

Wolfgang Schmidt and his mathematics[1]

We would like to give a brief survey on certain results and problems related to the works of Wolfgang Schmidt. Of course it is not possible in 40 minutes to discuss all the important results by Wolfgang Schmidt and their developments. We will concentrate ourselves on the following topics:

- Minkowski-Hlawka theorem, packing problems and the results by Schmidt and Rogers;
- Schmidt's (α, β)-games and badly approximable numbers;
- Mildly restricted Diophantine approximations;
- One-parameter families of lattices and their applications.

[1]Supported by RFBR grant No. 12-01-00681-a and by the grant of Russian Government, project 11. G34.31.0053.

Yuri Nesterenko (Moscow, Russia)

Some identities of Ramanujan type

The Eisenshten series $E_{2k}(\tau), k \geqslant 1$, can be defined as

$$E_{2k}(\tau) = 1 + \frac{4k}{B_{2k}} \sum_{n=1}^{\infty} \sigma_{2k-1}(n) q^n, \qquad q = e^{2\pi i \tau}, \quad \tau \in \mathbb{C}, \, \Im\tau > 0,$$

where B_{2k} are Bernoulli numbers and $\sigma_m(n) = \sum_{d|n} d^m$. It is well known that the functions

$$e^{\pi i \tau}, E_2(\tau), E_4(\tau), E_6(\tau)$$

are algebraically independent over $\mathbb{C}$ but for any imaginary quadratic ξ, $\Im\xi > 0$, the field generated over $\mathbb{Q}$ by three numbers $E_2(\xi), E_4(\xi), E_6(\xi)$ has transcendence degree 2. This degeneracy allowed to prove algebraic independence of the numbers π, e^{π} and $\Gamma(1/4)$ (with $\xi = i$).

We plan to discuss properties of functions

$$g_{u,v}(\tau) = \sum_{n=1}^{\infty} n^u \sigma_{-v}(n) q^n, \qquad 0 \leqslant u < v.$$

that are algebraically independent together with $e^{\pi i \tau}, E_2(\tau), E_4(\tau), E_6(\tau)$ over $\mathbb{C}$. Some identities of modular type are proved for these functions. As a consequence we prove that the transcendence degree of the field generated by the values of $g_{u,v}(\xi)$ is smaller that it is expected. The interest to these functions is motivated by equalities of the type

$$\zeta(3) = \frac{7\pi^3}{180} - 2\sum_{n=1}^{\infty} \sigma_{-3}(n) e^{-2\pi n}.$$

Similar formulas hold for other odd Riemann zeta-values. For the first time they were announced in 1901 by M. Lerch. They can be deduced from some functional identity announced in 1916 by S. Ramanujan and have been proved by E. Grosswald in 1970.

Ryotaro Okazaki (Kyoto, Japan)

A refinement on a Theorem of Nagell on totally imaginary quartic Thue equations

Let F be a binary quartic form with coefficients in $\mathbb{Z}$, where we assume $F(X, 1)$ has four imaginary roots. We consider the Thue equation $F(x, y) = 1$ in $x, y \in \mathbb{Z}$. Let $N(F)$ be one half of the number of solutions to our Thue equation.

Nagell investigated $N(F)$ under the assumption that the field defined by $F(X, 1)$ contains a quadratic subfield. Nagell points out there is one such quartic form, say $F^{(E)}$, such that $N(F^{(E)}) = 4$. He also proves $N(F) \leqslant 3$ provided F is inequivalent with $F^{(E)}$ under $GL_2(\mathbb{Z})$. He identifies two infinite families of quartic forms F with $N(F) = 3$.

We refine this result of Nagell by eliminating the condition of quadratic subfield. We find another quartic form with $N(F) = 4$. We will identify all quartic forms with $N(F) = 3$ under the technical condition $D(F) \geqslant 10^5$, where $D(F)$ denotes the discriminant of F.

Alexandr Polyanskii (Moscow, Russia)

On the irrationality measure of certain numbers

The irrationality measure can be define for any irrational α as the least upper bound on the κ such that the inequality

$$\left|\alpha - \frac{p}{q}\right| < q^{-\kappa}$$

has infinitely many rational solutions p/q. The irrationality measure is denoted as $\mu(\alpha)$.

The non-quadraticity measure can be define for any α which isn't a root of quadratic equation as the least upper bound on the κ such that the inequality

$$|\alpha - \beta| < H^{-\kappa}(\beta)$$

has infinitely many solutions in quadratic irrationalities β. Here $H(\beta)$ is the height of the characteristic polynomial of β. The non-quadraticity measure is denoted as $\mu_2(\beta)$.

We are going to present upper bounds on the irrationality measure and non-quadraticity for the numbers

$$\alpha_k = \sqrt{2k-1}\,\mathrm{arctg}\frac{\sqrt{2k-1}}{k-1}, \text{ then } k = 2m, m \in \mathbb{N}.$$

Some of numerical results have been summarized in the table below:

k	$\mu(\alpha_k) \leqslant$	$\mu_2(\alpha_k) \leqslant$	k	$\mu(\alpha_k) \leqslant$	$\mu_2(\alpha_k) \leqslant$
2	4.60105...	———	8	3.66666...	14.37384...
4	3.94704...	44.87472...	10	3.60809...	12.28656...
6	3.76069...	19.19130...	12	3.56730...	11.11119...

In particular we prove that

$$\mu\left(\frac{\pi}{\sqrt{3}}\right) \leqslant 4.60105\ldots$$

Andrey Raigorodskii (Moscow, Russia)

From combinatorial geometry to Ramsey theory[1]

This talk is concerned with some problems lying on the edge of combinatorial geometry and Ramsey theory. We start by discussing two classical and closely connected problems — that of Borsuk and that of Nelson – Hadwiger. The first problem is in finding the value $f(n)$ equal to the minimum number f such that any bounded non-singleton set in $\mathbb{R}^n$ can be divided into f parts of smaller diameter. The second problem deals with the chromatic number $\chi(\mathbb{R}^n)$ of the Euclidean space, which is the smallest number of colors needed to paint all the points in $\mathbb{R}^n$ so that any two points at the distance 1 receive different colors.

The two problems admit graph theoretic interpretations that make them so close one to the other. In the case of Nelson – Hadwiger's problem, one studies the chromatic numbers of distance graphs $G = (V, E)$, where $V \subseteq \mathbb{R}^n$ and E consists of some of the pairs $(\mathbf{x}, \mathbf{y}) \in V \times V$ such that $|\mathbf{x} - \mathbf{y}| = 1$. In the case of Borsuk's problem, distance graphs are substituted by graphs of diameters. Here V is a (finite) set in $\mathbb{R}^n$ and vertices in V are joined by an edge if and only if they realize the diameter of V.

The above interpretations do not only provide a good language to speak about both problems, but also they give rise to a number of important questions. In particular, classical questions of Ramsey theory may be naturally asked about distance graphs and graphs of diameters.

In our talk, we shall first give a brief overview of Borsuk's and Nelson – Hadwiger's problems. Then we shall proceed to discussing Ramsey theoretic aspects of both problems. For example, we shall speak about graphs with small cliques and large chromatic numbers (small independence numbers) or about graphs with simultaneously high girth and high chromatic numbers.

[1] This work was supported by the grants of RFBR No. 09-01-00294 and of the President of the Russian Federation No. MD-8390.2010.1, NSH-691.2008.1.

Andrzej Schinzel (Warsaw, Poland)

A property of quasi-diagonal forms

Theorem. Let k be a positive integer, $l = 2k^2(k,2)^2 - k(k,2)$ and let $F_i(X_i)$ be forms in $Z[X_i]$ of degree k, where X_i $(i = 1, 2, ...)$ are disjoint vectors of variables. Assume that (*) either not all forms F_i are semi-definite of the same sign or at least $kl + 1$ forms are non-singular. Then, there exists a positive integer s_0 such that every integer represented over Z by the sum of $F_i(X_i)$ over i from 1 to s is represented over Z by a similar sum over i from 1 to s_0. If $k = 2$ the condition (*) can be omitted.

Denis Shatskov (Astrakhan, Russia)
On the irrationality measure function in average[1]

Let α be a real irrational number. We consider the function

$$\psi_\alpha(t) = \min_{1\leqslant q\leqslant t} ||q\alpha||$$

(here q is an integer number and $\|\cdot\|$ stands for the distance to the nearest integer). We study the integral

$$I_\alpha(t) = \int\limits_1^t \psi_\alpha(\xi)d\xi.$$

Theorem 1. *For almost all* (*in sense of Lebesgue measure*) *numbers* α *one has*

1) $\lim\limits_{t\to\infty} \frac{I_\alpha(t)}{N(\alpha,t)} = \frac{1}{2}$,

2) $\lim\limits_{t\to\infty} \frac{I_\alpha(t)}{\ln t} = \frac{6\ln 2}{\pi^2}$.

Given $t > 1$ we define $N = N(\alpha, t)$ by the conditions $q_N \leqslant t < q_{N+1}$.

In the next two theorem we have calculated the extremal values for the quantity $\frac{I_\alpha(t)}{N(\alpha,t)}$.

Theorem 2. *For every irrational number* $\alpha \in (0; 1)$ *one has*

1) $\limsup\limits_{t\to\infty} \frac{I_\alpha(t)}{N(\alpha,t)} \leqslant 1$,

2) $\liminf\limits_{t\to\infty} \frac{I_\alpha(t)}{N(\alpha,t)} \geqslant \frac{1}{2} - \frac{\sqrt{5}}{10}$.

The bounds from Theorem 2 are optimal ones.

Theorem 3. *Given* $d \in \left[\frac{1}{2} - \frac{\sqrt{5}}{10}; 1\right]$ *there exists* α*, such that*

$$\lim_{t\to\infty} \frac{I_\alpha(t)}{N(\alpha,t)} = d.$$

It is proved that for any two real numbers α and β such that $\alpha \pm \beta \notin \mathbb{Z}$, the difference function

$$\psi_\alpha(t) - \psi_\beta(t)$$

changes its sign infinitely many often as $t \to \infty$.

[1] Research is supported by the grant RFBR No. 12-01-00681-a.

Easily to find two algebraically independent real numbers α and β such that the limits

$$\lim_{t\to\infty} \frac{I_\alpha(t)}{\ln t} \quad \text{and} \quad \lim_{t\to\infty} \frac{I_\beta(t)}{\ln t}$$

are different. This shows that there is no general oscillating property for the difference function $I_\alpha(t) - I_\beta(t)$.

Iekata Shiokawa (Yokohama, Japan)

Algebraic relations for the sixteen Eisenstein series

We study sixteen Eisenstein series generated by Fourier expansions of the Jacobian elliptic functions or some of their squares, which have been discussed by Jacobi, Ramanujan, Zucker, and many others. We determine all the sets of series belonging to these sixteen parametric families of q-series such that the values at each algebraic point of the series in the set are algebraically independent over the rationals. It is known that any four of these series are algebraically dependent over the rationals. We give explicit algebraic relations for all two or three of these series which are algebraically dependent.

Ilya Shkredov (Moscow, Russia)
Additive structures in multiplicative subgroups

We investigate various "random" properties of multiplicative subgroups of finite fields such as intersections of such subgroups with its additive shifts, basis properties, representations of subgroups as sumsets, intersections with arithmetic progressions and so on. We give a survey of old and new results in the field.

Anton Shutov (Vladimir, Russia)
Multidimensional Hecke-Kesten problem

Let $\alpha = (\alpha_1, \ldots, \alpha_N)$, $a = (a_1, \ldots, a_N)$ be two N-dimensional vectors. Consider a toric shift

$$R_\alpha : x \to x + \alpha \pmod{\mathbb{Z}^N}.$$

Suppose that $1, \alpha_1, \ldots, \alpha_N$ are linearly independent over $\mathbb{Z}$. Then from the Weyl theorem follows that the points $\{T^n(a)\}$ are uniformly distributed on N-dimensional torus $\mathbb{T}^N = \mathbb{R}^N/\mathbb{Z}^N$ for any a. Consider the remainder term of the uniform distribution theorem:

$$r(\alpha, X, n) = \sup_a |\sharp\{k : 0 \leqslant k < n, R_\alpha^k(a) \in X\} - n|X||,$$

where $X \subset \mathbb{T}^N$ is a domain and the characteristic function of X is integrable in the sense of Riemann.

The Hecke-Kesten problem is to find all triples (α, X, C) such that

$$r(\alpha, X, n) \leqslant C$$

for any n. Sets X with this property are called bounded remainder sets for the shift R_α.

Theorem 1. *Consider a partition of N-dimensional torus*

$$\mathbb{T}^N = \bigsqcup_{i=0}^{N} \bigsqcup_{j=1}^{k_i} E_i(j)$$

such that

$$R_\alpha(E_i(j)) = E_i(j+1)$$

for $0 \leqslant i \leqslant N$, $1 \leqslant j \leqslant k_i - 1$, and

$$R_\alpha\left(\bigsqcup_{i=1}^{N} E_i(k_j)\right) = \bigsqcup_{i=1}^{N} E_i(0).$$

Then all the sets $E_i(j)$ are bounded remainder sets for R_α with the effectively computable constants $C = C(\alpha, E_i(j))$.

These construction generalizes earlier constructions of bounded remainder sets given by Hecke, Rauzy, Balladi and Rockmore, Shutov, Zhuravlev and others.

In the talk we also discuss three special cases of the theorem 1:

- one-dimensional generalized Fibonacci tiling;

- generalized Rauzy fractals;

- partitions, obtained by $(N+1)$-domain exchange transformations on $\mathbb{T}^N$.

It is interesting that each set $E_i(j)$ is a bounded remainder set for an infinite family of toric shifts.

Theorem 2. *Under the conditions of the theorem 1 the sets $E_i(j)$ are also bounded remainder sets for the shifts R_β with*

$$\beta = \frac{1}{h}(\alpha + b),$$

and $h \in \mathbb{Z}$, $b \in \mathbb{Z}^N$. Moreover,

$$C(\beta, E_i(j)) \leqslant C(\alpha, E_i(j))|h|.$$

Theorem 2 can be generalized on arbitrary bounded remainder sets, not only obtained by the theorem 1.

We also discuss some improvements of the estimate of $C(\beta, E_i(j))$ in the special case $k_0 = k_1 = \ldots = k_N$.

Keijo Väänänen (Oulu, Finland)

On arithmetic properties of some q-series

We shall consider linear independence of the values of certain q-series, which are special cases of the function

$$f_q(z) = \sum_{n=0}^{\infty} \left(\prod_{k=0}^{n-1} \frac{M(q^k)}{N(q^k)} \right) z^n,$$

where $M(x)$ and $N(x)$ are polynomials with $N(0) = 1$ and $|q| < 1$.

Carlo Viola (Pisa, Italy)

Towards simultaneous approximations to $\zeta(2)$ and $\zeta(3)$

If in a triple integral over $(0,1)^3$ yielding a Q-linear form in 1 and $2\zeta(3)$ we replace a suitable integration over $(0,1)$ with a contour integral, we obtain a Q-linear form in 1 and $\zeta(2)$ where the coefficient of $\zeta(2)$ coincides with the coefficient of $2\zeta(3)$ in the previous linear form. Thus such a construction yields simultaneous rational approximations to $\zeta(2)$ and $\zeta(3)$, though not strong enough to prove the Q-linear independence of 1, $\zeta(2)$ and $\zeta(3)$.

Michel Waldschmidt (Paris, France)

Families of Diophantine equations without non-trivial solutions

In joint works with Claude Levesque, we give new examples of families of Diophantine equations having only trivial solutions. Our first results relied on Schmidt's subspace theorem, and therefore were not effective. We are now able to produce effective results in some cases.

Barak Weiss (Be'er Sheva, Israel)

The Mordell-Gruber spectrum and homogeneous dynamics

Given a lattice in euclidean space, its Mordell constant is the supremum of the volumes (suitably normalized) of central symmetric boxes with sides parallel to the axes, containing no nonzero lattice points. This is a well-studied quantity in the geometry of numbers, and is invariant under the action of diagonal matrices on the space of lattices. In joint work with Uri Shapira, using dynamics of the diagonal group we obtain new results about the set of possible values of the Mordell constant (the so-called Mordell-Gruber spectrum).

Victoria Zhuravleva (Moscow, Russia)
Diophantine approximation with Fibonacci numbers

The Fibonacci sequence F_n is known from ancient times, but still hides many interesting properties. In this talk we discuss new results which concern the field of Diophantine approximations. Let α be a real number, while φ represent the golden ratio. We introduce the following inequalities:

1) $\inf\limits_{n\in\mathbb{N}} ||F_n\alpha|| \leqslant \frac{\varphi-1}{\varphi+2}$, 2) $\liminf\limits_{n\to\infty} ||F_n\alpha|| \leqslant \frac{1}{5}$, 3) $\liminf\limits_{n\to\infty} ||\varphi^n\alpha|| \leqslant \frac{1}{5}$.

These bounds cannot be improved upon.