ONLINE CONFERENCE

Diophantine Analysis
and
Related Topics

Zoom, May 31 - June 04, 2021

Program and Abstract Book

The lectures are scheduled with respect to Moscow time (GMT+3)

Schedule

Monday 31.05

OPENING of the CONFERENCE at 10:45

11:00 - 11:30	Victor Beresnevich	Khintchine's theorem for manifolds: 'hypotheses non fingo'
11:45 - 12.15	Elio Joseph	Rational approximation of linear subspaces
12:30 - 13:00	Arturas Dubickas	Transcendency of some constants related to integer sequences of polynomial iterations
13:15 - 13:45	Nikita Shulga	Rational approximation to two irrational numbers
	Break	
17:00 - 17:30	Vasilii Bernik and Nikolay Kalosha	Metric Theory of Diophantine Approximation and Distribution of Real Algebraic Numbers
17:45 - 18:15	Cameron Stewart	On Tijdeman's theorem and generalizations
18:30 - 19:00	Anton Mosunov	On the Representation of Integers by Binary Forms Associated with Algebraic Trigonometric Quantities

Tuesday 01.06

11:00 - 11:30	Dmitrii Badziahin	Uniform simultaneous exponent of Diophantine approximation of numbers and their consecutive powers
11:45 - 12.15	Faustin Adiceam	Bad approximability in Diophantine Approximation
12:30 - 13:00	Demi Allen	Dyadic approximation in the middle-third Cantor set
13:15 - 13:45	Sam Chow	Inhomogeneous Littlewood and Duffin–Schaeffer
	Break	
17:00 - 17:30	Jörg Thuswaldner	On the second Lyapunov exponent of some multidimensional continued fraction algorithms
17:45 - 18:15	Alan Haynes	Uniform labeling and quasiperiodicity for Steinhaus problems in algebraic number fields
18:30 - 19:00	Florin Boca	Distribution of reduced quadratic irrationals

Wednesday 02.06

11:00 - 11:30	Alexei Ustinov	Tropical Somos sequences and billiards
11:45 - 12.15	Antoine Marnat	Dirichlet is not just Bad and Singular
12:30 - 13:00	Johannes Schleischitz	A phenomenon in p-adic approximation
13:30 - 14:00	Tanguy Rivoal	Siegel's problem for E-functions
	Break	
17:00 - 17:30	Anton Shutov	Some results associated with Fibonacci numeration
17:45 - 18:15	Lenny Fukshansky	On sparse geometry of numbers
	19:00 Virtual conference party	

Thursday 03.06

11:00 - 11:30	Alexander Gorodnik	Multiple equidistribution of measures
11:45 - 12.15	Erez Nesharim	Badly approximable vectors on curves
12:30 - 13:00	Barak Weiss	Geometric and arithmetic aspects of approximation vectors 1
13:15 - 13:45	Uri Shapira	Geometric and arithmetic aspects of approximation vectors 2
14:00 - 14:30	Omri Solan	Parametric geometry of numbers with general flows
	Break	
17:00 - 17:30	Andreas Strömbergsson	Towards a zero-one law for improvements to Dirichlet's Theorem in general dimension
17:45 - 18:15	Dmitrii Kleinbock	On Dirichlet improvable pairs with respect to arbitrary norm
18:30 - 19:00	Lei Yang	Multiplicative Diophantine approximation on planar lines and effective equidistribution

Friday 04.06

11:00 - 11:30	Javier Fresán	A non-hypergeometric E-function
11:45 - 12.15	Peter Jossen	Roots of E-functions
12:30 - 13:00	Mariya Bashmakova, Vladislav Salikhov	On rational approximations for some values of $\arctan \frac{s}{r}, s, r \in \mathbb{N}, s < r$
13:15 - 13:45	Stéphane Fischler	Effective algebraic independence of values of E-functions
	Break	
17:00 - 17:30	Raffaele Marcovecchio	A linear independence criterion for Hermite-Padé approximations of type II
17:45 - 18:15	Paul Voutier	Bounds for the number of squares in binary recurrence sequences

ABSTRACTS

Faustin Adiceam, **Bad approximability in Diophantine Approximation**

Abstract: After surveying recent advances in the theory of badly approximable vectors, we will introduce the concept of vectors with a given uniform Diophantine type. These arise naturally in problems in Diophantine geometry but are more delicate to deal with with the classical theories developed so far to study the concept of bad approximability.

Based on joint work with Yaar Solomon and Barak Weiss.

Demi Allen, **Dyadic approximation in the middle-third Cantor set**

Abstract: In 1984, Mahler asked how well irrational numbers in the middle-third Cantor set can be approximated:

(i) by rational numbers in the Cantor set, and

(ii) by rational numbers not in the Cantor set.

This question generated great interest among a number of mathematicians and nowadays the topic of Diophantine Approximation on Fractals in general is a very popular one. In this talk, I will discuss a recent contribution to this topic, and to Mahler's original question, obtained jointly with Sam Chow (Warwick) and Han Yu (Cambridge). In particular, I will talk about the problem of approximating points in the Cantor set by dyadic rationals (that is, rationals which have denominators which are powers of 2). I will discuss a conjecture on this topic due to Velani, some progress towards this conjecture, and why studying dyadic approximation in the middle-third Cantor set is a difficult problem.

Dzmitry Badziahin, **Uniform simultaneous exponent of Diophantine approximation of numbers and their consecutive powers**

For a transcendental number $\xi \in \mathbb{R}$, its exponent $\hat{\lambda}_n(\xi)$ of Diophantine approximation is defined as the supremum of all $\lambda > 0$ such that the system of inequalities

$$\max_{1\le i\le n} ||q\xi^i|| \le Q^{-\lambda}, \quad 1 \le q \le Q$$

has a solution $q \in \mathbb{Z}$ for all large values of Q. It is one of the most mysterious exponents of Diophantine approximation. For example, for $n \ge 3$ we even do not know if it can take any value apart from $1/n$.

I will talk about recent developments in an understanding of $\hat{\lambda}_n(\xi)$. In particular, we will slightly improve the upper bounds for $\hat{\lambda}_n(\xi)$ for even values of n and will introduce non-trivial relations between $\hat{\lambda}_n(\xi)$ and other exponents of Diophantine approximations for ξ.

Mariya G. Bashmakova, Vladislav Salikhov, On rational approximations for some values of $\arctan\frac{s}{r}, s, r \in \mathbb{N}, s < r$

Abstract: In the talk we present various new results for irrationality measure of some values of $\arctan x$. Initially we applied symmetric complex integrals to approximate values of form $\arctan\frac{1}{n}, n \in \mathbb{N}$. This method gave us new estimates for $\arctan\frac{1}{2}, \arctan\frac{1}{3}$ but not for other ones, so we changed main construction.

We considered new integral without property of symmetry of integrand, which based on idea of Q.Wu. Integral construction of this type at first allowed us to improve estimates for irrationality measure of $\arctan\frac{1}{5}$, $\arctan\frac{1}{6}$, $\arctan\frac{1}{10}$. Best previous results for these values were obtained by E.B.Tomashevskaya, who also used in her work symmetric complex integrals.Then we generalized this idea and built common integral for estimation of some values of form $\arctan\frac{s}{r}$.

This new integral

$$Y_n =$$

$$= \frac{1}{i}\int_{r-si}^{r+si} \frac{(x-r-si)^{(1-\alpha-\beta)n}(x-r+si)^{(1-\alpha-\beta)n}((x-r)(rx-(r^2+s^2))^{\alpha n}(kx^2-(2kr+1)x+k(r^2+s^2))^{\beta n}dx}{x^{n+1}}$$

$$n, s, r, k \in \mathbb{N}, s < r, \alpha, \beta \in \mathbb{R}, \alpha n, \beta n \in \mathbb{N}.$$

gave us a number of new estimates for extent of irrationality, in particular for $\arctan\frac{1}{7}$, $\arctan\frac{1}{9}$, $\arctan\frac{2}{9}$, $\arctan\frac{2}{11}$ and others.

Victor Beresnevich, Khintchine's theorem for manifolds: 'hypotheses non fingo'

Abstract: In this talk I will describe a recent joint work with Lei Yang on Khintchine's theorem for manifolds which tackles a problem posed by Kleinbock and Margulis in 1998. I will also describe its generalisation using Hausdorff measure and dimension which enables us to make progress in discovering the spectrum of the exponent $\lambda_n(x)$ of simultaneous rational approximations to $(x, x^2, \ldots, x^n)$ for real irrational numbers x.

Vasilii Bernik, Nikolay Kalosha, Metric Theory of Diophantine Approximation and Distribution of Real Algebraic Numbers

Abstract: It is well known that the set of algebraic numbers is everywhere dense on the real line and the complex plane. However, if we consider a natural ordering of algebraic numbers of degree n and height bounded from above by Q, their distribution isn't uniform. Moreover, one can construct intervals of size $c(n)Q^{-1}$ that are free of such algebraic numbers.

We present both established and recent results in Diophantine approximation related to the distribution of algebraic numbers, in particular a recent result describing the number of algebraic numbers of fixed degree and bounded height contained in small intervals of the real line.

Florin Boca, **Distribution of reduced quadratic irrationals**

Abstract: Reduced quadratic irrationals (QIs) arising from the regular CF are closely related with the Pell equation and with closed geodesics on the modular surface. By classical work of Pollicott, they are known to be equidistributed with respect to the Gauss probability measure, when ordered by their corresponding closed primitive geodesic length. An effective version of this result was established by Ustinov. This talk will consider the reduced QIs arising from the even CF (Schweiger), the odd CF (Rieger), and the backward CF (Renyi), where analogous (effective) equidistribution results with respect to their invariant measures have been recently proved. This is joint work with M. Siskaki.

Sam Chow, **Inhomogeneous Littlewood and Duffin–Schaeffer**

Abstract. Gallagher's theorem describes the multiplicative diophantine approximation rate of a typical vector. We establish a completely inhomogeneous version of Gallagher's theorem, a diophantine fibre refinement, and an unexpected threshold on Liouville fibres. Along the way, we prove an inhomogeneous version of the Duffin-Schaeffer conjecture for a class of non-monotonic approximation functions. Joint with Niclas Technau.

Artūras Dubickas, **Transcendency of some constants related to integer sequences of polynomial iterations**

Abstract: Let

$$P(x) = a_0x^d + a_1x^{d-1} + \cdots + a_d$$

be a polynomial with rational coefficients of degree $d \geq 2$, and let x_n, $n = 0, 1, 2, \ldots$, be a sequence of integers satisfying $x_{n+1} = P(x_n)$ for $n \geq 0$ and $x_n \to \infty$ as $n \to \infty$. Then, by a recent result of Wagner and Ziegler, the limit

$$\alpha = \lim_{n\to\infty} x_n^{d^{-n}} > 1$$

is either an integer or an irrational number, and x_n is approximately $a_0^{-1/(d-1)}\alpha^{d^n} - a_1/(da_0)$.

Under assumption that $a_0^{1/(d-1)}$ is rational (which is always true for $d = 2$), we completely characterize all the cases when the limit α is an algebraic number. Our results imply that α can be an integer, a quadratic Pisot unit with α^{-1} being its conjugate over the rationals, or a transcendental number. In most cases α is transcendental. For each $d \geq 2$ all the polynomials P of degree d for which α is an integer or a quadratic Pisot unit are described explicitly. The main theorem implies that several constants related to sequences that appear in a paper of Aho and Sloane and in the online Encyclopedia of Integer Sequences (OEIS) are not just irrational (as was shown by Wagner and Ziegler), but transcendental. The paper will appear in *The Ramanujan Journal*.

Stéphane Fischler, **Effective algebraic independence of values of E-functions**

Abstract: E-functions have been introduced by Siegel in 1929: this is a class of functions that contain, among others, the exponential function and Bessel functions. Given a finite family of algebraically independent E-functions, we consider the set S of algebraic points at which their values are algebraically dependent. The Siegel-Shidlovsky theorem (proved in 1955 and refined since then by several authors) allows one to show that S is finite. The aim of this lecture is to give an algorithm to determine this set S. This is a joint work with Tanguy Rivoal.

Javier Fresán, **A non-hypergeometric E-function**

Abstract: In a landmark 1929 paper, Siegel introduced the class of E-functions with the goal of generalising the transcendence theorems for the values of the exponential. E-functions are power series with algebraic coefficients subject to certain growth conditions that satisfy a linear differential equation. Besides the exponential, examples include Bessel functions and a rich family of hypergeometric series. Siegel asked whether all E-functions are polynomial expressions in these hypergeometric series. I will explain why the answer is negative (joint work with Peter Jossen).

Lenny Fukshansky, **On sparse geometry of numbers**

Abstract: Let L be a lattice of full rank in n-dimensional real space. A vector in L is called i-sparse if it has no more than i nonzero coordinates. We define the i-th successive sparsity level of L, $s_i(L)$, to be the minimal s so that L has s linearly independent i-sparse vectors, then $s_i(L) \leq n$ for each $1 \leq i \leq n$. We investigate sufficient conditions for $s_i(L)$ to be smaller than n and obtain explicit bounds on the sup-norms of the corresponding linearly independent sparse vectors in L. This result can be viewed as a partial sparse analogue of Minkowski's successive minima theorem. We then use this result to study virtually rectangular lattices, establishing conditions for the lattice to be virtually rectangular and determining the index of a rectangular sublattice. We further investigate the 2-dimensional situation, showing that virtually rectangular lattices in the plane correspond to elliptic curves isogenous to those with real j-invariant. We also identify planar virtually rectangular lattices in terms of a natural rationality condition of the geodesics on the modular curve carrying the corresponding points. This is joint work with Pavel Guerzhoy and Stefan Kuehnlein.

Alexander Gorodnik, **Multiple equidistribution of measures**

Abstract: We investigate behavior of certain measures arising in multiplicative Diophantine approximation and establish asymptotic estimates on their correlations. This is a joint work with M. Björklund

Alan Haynes, **Uniform labeling and quasiperiodicity for Steinhaus problems in algebraic number fields**

Abstract: We consider a higher dimensional version of the Steinhaus problem (three gaps theorem) involving the fractional parts of a linear form in more than one variable. We focus on the case when the coefficients of the linear form in question form a basis for an algebraic number field. In this case, we show that there is a fixed finite set S such that every 'gap' that appears can be rescaled by a unit in the field to become an element of S. In terms of this uniform labeling of the gaps by elements of S we are able to describe the frequencies at which various gaps occur as a "quasiperiodic motion," up to an exponentially small error term.

Elio Joseph, **Rational approximation of linear subspaces**

Abstract: Let A and B be two subspaces of $\mathbb{R}^n$ of respective dimensions d and e with $d+e \leqslant n$. The proximity between A and B is measured by $t = \min(d,e)$ canonical angles $0 \leqslant \theta_1 \leqslant \cdots \leqslant \theta_t \leqslant \pi/2$; we set $\psi_j(A,B) = \sin\theta_j$. If B is a rational subspace, his complexity is measured by its height $H(B) = \mathrm{covol}(B \cap \mathbb{Z}^n)$. We are interested in the exponent of approximation $\mu_n(A|e)_j$ defined as the upper bound (possibly equal to $+\infty$) of the set of $\beta > 0$ such that the inequality $\psi_j(A,B) \leqslant H(B)^{-\beta}$ holds for infinitely many rational subspaces B of dimension e. In this talk, we will focus on the approximation of planes by rational planes in $\mathbb{R}^4$. More precisely, we will show that the minimal value taken by $\mu_4(A|2)_1$ is 3 when A ranges through the set of planes of $\mathbb{R}^4$ such that for all rational planes B one has $A \cap B = \{0\}$.

Peter Jossen, **Roots of E-functions**

Abstract: I will discuss a few observations and questions concerning the distribution of roots of E-functions in the complex plane. All information about these roots can be encoded in a zeta function. In special cases, when the E-function is some Bessel function, Airy function or a trigonometric polynomial, one recovers so-called "spectral" zeta functions of operators studied by physicists. I will show, mostly by means of examples, how to calculate the values of these zeta functions at positive integers or, more interestingly, at negative integers.

Dmitrii Kleinbock, **On Dirichlet Improvable pairs with respect to arbitrary norm**

Abstract: In 2013 Akhunzhanov and Shatskov proved that for any $c \in [0, 2/\sqrt{3}]$ there exists $\boldsymbol{x} \in \mathbb{R}^2$ with

$$c_E(\boldsymbol{x}) := \limsup_{t\to\infty} \; t \cdot \left(\min_{1\leq q\leq t} \mathrm{dist}_E(q\boldsymbol{x}, \mathbb{Z}^2) \right)^2 = c,$$

where 'dist_E' is the Euclidean distance in $\mathbb{R}^2$. Similarly one can define $c_\nu(\boldsymbol{x})$ with respect to any norm ν on $\mathbb{R}^2$. It is not hard to see that almost surely $c_\nu(\boldsymbol{x})$ attains its maximal value $=: c_\nu$, that is, almost every $\boldsymbol{x}$ is *not ν-Dirichlet-improvable*. We prove that the set $\{\boldsymbol{x} \in \mathbb{R}^2 : c_\nu(\boldsymbol{x}) < c_\nu\}$ of ν-Dirichlet-improvable pairs is

hyperplane absolute winning, in particular has Hausdorff dimension 2. For the supremum norm this is due to Davenport and Schmidt. Our proof is dynamical and consists of constructing an exceptional set of orbits in the space of lattices. Joint work with Anurag Rao.

Raffaele Marcovecchio A linear independence criterion for Hermite-Padé approximations of type II

Abstract: The classical linear independence criterion for Hermite–Padé approximations of type II yields the linear independence of $m+1$ real numbers $1, \gamma_1, \ldots, \gamma_m$ under suitable assumptions on the approximations $q_n\gamma_j - p_n^{(j)}$ (for $j = 1, \ldots, m$ and $n \to \infty$) with suitably chosen (or constructed) integral coefficients q_n, $p_n^{(1)}, \ldots, p_n^{(m)}$. Until recently, no partial result was known, as far as I know, for approximations that do not converge to zero. I present a new criterion that copes with this difficulty, and yields a lower bound of the dimension of the vector space spanned over $\mathbb{Q}$ by $1, \gamma_1, \ldots, \gamma_m$ under natural assumptions on a *vector* of such approximations. I also sketch some applications of this criterion.

Antoine Marnat, Dirichlet is not just Bad and Singular

Abstract: It is well known that in dimension one, the set of Dirichlet improvable real numbers consists precisely of badly approximable and singular numbers. We show that in higher dimensions this disjoint union is not the full set of Dirichlet improvable vectors: we prove that there exist uncountably many Dirichlet improvable vectors that are neither badly approximable nor singular. We construct these numbers using the parametric geometry of numbers. Furthermore, by doing so we can choose the exponent of Diophantine approximation by a rational subspace of dimension exactly d with d between 0 and n-1.

Anton Mosunov, On the Representation of Integers by Binary Forms Associated with Algebraic Trigonometric Quantities

Abstract: Let $F(x, y)$ be a binary form of degree $d \geq 3$ and non-zero discriminant. In 2019, Stewart and Xiao proved that the number of integers of absolute value at most h which are represented by the form F is asymptotic to $C_F h^{2/d}$, where C_F is a positive real number which depends only on F. We estimate C_F for four families of binary forms. The first two families that we are interested in are homogenizations of the minimal polynomials of $2\cos(2\pi/n)$ and $2\sin(2\pi/n)$, which we denote by $\Psi_n(x, y)$ and $\Pi_n(x, y)$, respectively. The remaining two families of binary forms that we consider are homogenizations of Chebyshev polynomials of the first and second kinds, denoted $T_n(x, y)$ and $U_n(x, y)$, respectively.

Erez Nesharim, **Badly approximable vectors on curves**

Abstract: We study the set of badly approximable vectors on analytic nondegenerate curves. For that we use the Kleinbock–Lindenstrauss–Weiss uniform nondivergence estimates in the space of lattices and a variant of Schmidt's game played on the support of Ahlfors-regular measures on the real line. Applied to the curves $x \to (x, x^2, ..., x^n)$this proves the existence of (an absolute winning set of) real numbers which are badly approximable by algebraic numbers. This talk is based on joint works with Victor Beresnevich and Lei Yang.

Tanguy Rivoal, **Siegel's problem for E-functions**

Siegel asked in 1949 if every E-function can be written as a polynomial in hypergeometric E-functions. I will explain why a positive answer to this question contradicts a generalization to exponential periods of Grothendieck's periods conjecture. This is a joint work with Stéphane Fischler.

Johannes Schleischitz, **A phenomenon in p-adic approximation**

Abstract: As pointed out in a recent paper by de Mathan (2019), in contrast to the real case, for a p-adic number ξ there may be many good rational approximations x/y, i.e. with $|x/y - \xi|_p$ small compared to $\max\{|x|, |y|\}$, where $|x|$ and $|y|$ are not of the same magnitude. De Mathan was primarily interested in irrational quadratic (thus algebraic) $\xi \in \mathbb{Q}_p$. We investigate this phenomenon from a different point of view, which leads us to the study of new exponents of p-adic approximation. We present several results and open questions on these exponents. Joint work with Yann Bugeaud.

Uri Shapira, **Barak Weiss**, **Geometric and arithmetic aspects of approximation vectors 1,2**

Abstract: Associated to a vector theta in $\mathbb{R}^d$ one can define a sequences of best approximants and epsilon approximants, and one can ask various statistical questions about these approximations. For $d = 1$ these notions are classical and well-studied, and for $d > 1$ are coming under increased attention in recent years. We discuss the following invariants attached to an approximation vector (p, q): the displacement vector (size and direction of the difference $q\theta - p$); the geometry of the quotient lattice (projection of $\mathbb{Z}^{d+1}$ modulo the line through (p, q)); the arithmetic properties of (p, q). One can ask about the typical behavior of these invariants, for given theta, along the sequence of approximations (p_k, q_k), in the limit as k goes to infinity. We answer these questions by constructing a section to a one parameter flow on an adelic cover of the space of lattices, and relating the invariants mentioned above to hitting points on our section.

Nikita Shulga, Rational approximations to two irrational numbers

Abstract: For irrational number ξ we consider irrationality measure function $\psi_\xi(t) = \min_{1\leqslant q\leqslant t, q\in\mathbb{Z}} ||q\xi||$, where $||\cdot||$ - distance to the nearest integer. In this talk we give a result on a difference of these functions for two irrational numbers α and β for which $\alpha \pm \beta \notin \mathbb{Z}$, namely

$$\left|\frac{1}{\psi_\alpha(t)} - \frac{1}{\psi_\beta(t)}\right| \geqslant Ct \quad \text{for infinitely many } t.$$

Constant C is equal to $C = \sqrt{5}(1-\sqrt{\phi}) = 0.47818^+$, where $\phi = \frac{\sqrt{5}-1}{2}$.
We also show that constant C is optimal.
This result improves previously known results by Moshchevitin and Dubickas.

Anton Shutov, Some results associated with Fibonacci numeration

Abstract: Let $\{F_n\}$ be a sequence of Fibonacci numbers, i.e. $F_{n+1} = F_n + F_{n-1}$ and $F_1 = F_2 = 1$. Then any natural n can be represented as

$$n = \sum_{i=2}^{k(n)} f_i(n) F_i,$$

where $f_i(n) \in \{0,1\}$ and $f_i(n)f_{i-1}(n) = 0$. This representation is called an expansion of n in Fibonacci numeration system.

Let $w = w_l \dots w_1$ be some word over the alphabet $\{0,1\}$. Suppose that w does not contain subword 11. In this case we will say that the word w is admissible. Define the set

$$\mathbb{N}(w) = \{n \in \mathbb{N} : f_2(n) = w_1, \dots, f_{l+1}(n) = w_l\}.$$

In other words, $\mathbb{N}(w)$ is a set of natural numbers which expansions in Fibonacci numeration system end by the word w.

Theorem 1. *For any admissble word w there exists a segment $I(w) \in \mathbb{R}/\mathbb{Z}$ such that $n \in \mathbb{N}(w)$ if and only if $\{n\tau\} \in I(w)$. Here $\tau = \frac{\sqrt{5}+1}{2}$. The length of $I(w)$ is τ^{-l} if w begins with 0 and the length is $\tau^{-(l+1)}$ if w begins with 1.*

Corollary *For any sequence $\{a_n\}$ such that $\{a_n\tau\}$ is uniformly distributed modulo one, $\mathbb{N}(w)$ contains infinitely many terms of $\{a_n\}$.*

For example, for any admissible w $\mathbb{N}(w)$ contain infinitely many primes.

Let $r_k(n)$ be a number of solutions of the equation

$$n_1 + \ldots + n_k = n$$

with the condition $n_i \in \mathbb{N}(w)$ for all i.

Theorem 2. *We have*

$$r_k(x) \sim c_k(\{n\tau\})n^{k-1} + O(n^{k-2}\log n),$$

where $c_k(x)$ is some effectively computated continiou function. Moreover, $c_k(x)$ is piecewise polynomial of degree $k-1$. Also, if $k|I(w)| > 1$, there exists $n_0(w)$ such that $r_k(n) > 0$ for $n > n_0(w)$.

Further, consider the sets

$$\mathbb{F}_i(n) = \{n : \sum_{k=2}^{k(n)} f_k(n) \equiv 0 \bmod 2\}$$

and functions

$$F_{i,j}(X) = \sharp\{n < X : n \in \mathbb{F}_i, n+1 \in \mathbb{F}_j\}$$

Theorem 3. *We have*

$$F_{i,j}(X) = C_{i,j}X + O(\log X),$$

where

$$C_{i,j} = \begin{cases} \frac{\sqrt{5}}{10} & \text{if } i = j \\ \frac{5-\sqrt{5}}{10} & \text{if } i \neq j \end{cases}.$$

For binary numeration system similar result was independently obtained by Mahler and Eminyan.

These theorems are generalized in two different ways.

First, we can choose an arbitrary irrational α and consider Ostrwowski expansions on the sequence $\{Q_n\}$ of denominators of partial convergents to α. In this case the sets $I(w)$ are also intervals.

Second, we can consider expansions on more general linear recurrent sequences $\{T_n\}$:

$$T_n = a_1 T_{n-1} + a_2 T_{n-2} + \ldots + a_d T_{n-d}, \quad a_1 \geq a_2 \geq \ldots \geq a_{d-1} \geq a_d = 1.$$

In this case we must use multidimensional Kronecker sequences and analogues of the sets $I(w)$ can be described in the terms of Rauzy fractals.

Research is supported by the Russian Science Foundation (project 19-11-00065)

Omri Solan, **Parametric geometry of numbers with general flows**

Abstract: Parametric geometry of numbers is the problem of classifying the geometry of possible diagonal orbits in the space of lattices. Let g_t be a diagonal one parameter subgroup of $\mathrm{SL}_n(\mathbb{R})$. Let $g_t\Lambda$ be an orbit in the lattice space, and consider it's n Minkowski Minima, $\lambda_i(g_t\Lambda)$. Up to an additive constant, what kind of maps $i, t \mapsto \lambda_i(g_t\Lambda)$ can we attain? Schmidt, Summerer and Roy parametrized the set of such maps for some specific cases of g_t. We will see a combinatorial answer to this problem in its full generality and discuss the Hausdorff dimension related generalizations.

Cameron Stewart, **On Tijdeman's theorem and generalizations**

Abstract: In the 1970's Tijdeman showed that the integers generated multiplicatively from a finite set of primes are distributed in a regular fashion. We shall discuss the analogue of this result and some of its consequences where we replace a finite set of primes by a finite set of multiplicatively independent algebraic numbers.

Andreas Strömbergsson, **Towards a zero-one law for improvements to Dirichlet's Theorem in general dimension**

Abstract: Let ψ be a decreasing function defined on all large positive real numbers. We say that a real $m \times n$matrix Y is "ψ-Dirichlet" if for every sufficiently large real number t one can find integer vectors p $(m - \text{dim}) and$q(n-dim), with q non-zero, satisfying $|Yq - p|^m < \psi(t)$ and $|q|^n < t$ (where the bars denote supremum norm on vectors). This property was introduced by Kleinbock and Wadleigh in 2018, and it generalizes the property of Y being "Dirichlet improvable" which has been studied by several people, starting with Davenport and Schmidt in 1969. We will present results giving sufficient conditions on psi to ensure that the set of ψ-Dirichlet matrices Y has zero, resp., full measure.

Jörg Thuswaldner, **On the second Lyapunov exponent of some multidimensional continued fraction algorithms**

Abstract: In this talk we discuss the strong convergence of certain multidimensional continued fraction algorithms. In particular, in the two- and three-dimensional case, we are able to prove exponential convergence. For higher dimensions we provide heuristic results. These heuristics indicate that many classical multidimensional continued fraction algorithms cease to be strongly convergent for high dimensions. The only exception seems to be the Arnoux–Rauzy algorithm which, however, is defined only on a set of measure zero. This is joint work with V. Berthé and W. Steiner.

Alexey Ustinov, **Tropical Somos sequences and billiards**

Abstract: Somos-4 sequences are defined by a fourth-order quadratic recurrence relation of the form

$$s_{n+2}s_{n-2} = \alpha s_{n+1}s_{n-1} + \beta s_n^2.$$

The general solution of this quadratic recurrence relation has the form

$$s_n = AB^n \frac{\sigma(z_0 + nz)}{\sigma(z)^{n^2}},$$

where $\sigma(z) = \sigma(z; g_2, g_3)$ denotes the Weierstrass sigma function associated to the elliptic curve $y^2 = 4x^3 - g_2x - g_3$.

If $s_{-1} = u$, $s_0 = x$, $s_1 = y$, $s_2 = v$, $\alpha = uvxyw$ and $\beta = uvxyz$ then (Bykovskii, Ustinov, 2019) $s_n \in \mathbb{Z}[u, v, w, x, y, z]$. This is a generalization of the Laurent property of the Somos-4 proved by Fomin and Zelevinsky (2002). Some sequences (for example exponents of the variables u, v, x, y in s_n) satisfy the tropical version of Somos-4 recurrence

$$d_{n+2} + d_{n-2} = \max\{\alpha + d_{n+1} + d_{n-1}, \beta + 2d_n\}.$$

Many features of complex and real world become easily visible after tropicalization. In the tropical world oscillations of elliptic functions transform into the oscillations of billiard trajectories. The tropical simplicity

allows to guess and to predict properties of original objects. Hopefully this point of view will also work in the case of higher-order Somos sequences corresponding to higher genus curves.

Paul Voutier, **Bounds for the number of squares in binary recurrence sequences**

Abstract: We can define binary recurrence sequences $(x_k)_{k=-\infty}^{\infty}$ and $(y_k)_{k=-\infty}^{\infty}$ via the relationship

$$x_k + y_k\sqrt{d} = \left(a + b\sqrt{d}\right)\varepsilon^k,$$

where a, b and d are rational integers with $ab \neq 0$, d not a perfect square and ε is an irrational element of $\mathbb{Q}\left(\sqrt{d}\right)$.

When ε is a unit in the ring of integers of $\mathbb{Q}\left(\sqrt{d}\right)$, squares among the y_k's can give rise to solutions of diophantine equations of the form

$$X^2 - dY^4 = a^2 - b^2 d. \tag{1}$$

Furthermore, if the negative Pell equation $X^2 - dY^2 = -4$ is solvable, then we need only consider squares among the y_{2k}'s to investigate solutions of (1).

Using the hypergeometric method, I have developed a technique to bound the number of squares among the y_{2k}'s when $b = 1$, yielding results that are best possible for the number of squares in such sequences as well as for the number of solutions of the related equations in some cases. E.g.,

$$X^2 - \left(a^2 + p^m\right)Y^4 = -p^m.$$

In this talk, I discuss such results and what allows us to apply the hypergeometric method. I call this the representation lemma and I also discuss how this can be used to bound the number of squares in more general binary recurrence sequences.

Lei Yang, **Multiplicative Diophantine approximation on planar lines and effective equidistribution**

Abstract: We will study multiplicative Diophantine approximation property of typical points on planar lines. We will show that for any planar line, a strengthening of Littlewood's conjecture holds for almost every point on the line. The strengthening is sharp for typical points on the whole space. This is done by establishing an effective equidistribution result for certain one-parameter unipotent orbits in SL(3,R)/SL(3,Z). This is a joint work with Sam Chow.