On Diophantine exponents in dimension 4

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Let $\Theta = (\theta_1, ..., \theta_n), n \geqslant 2$ be a vector, we suppose that the numbers $1, \theta_1, ..., \theta_n$ are linearly independent over $\mathbb{Z}$. Put

$$\psi_\Theta(t) = \min_{q \in \mathbb{Z}_+, q \leqslant t} \max_{1 \leqslant j \leqslant n} ||q\theta_j||.$$

We consider the ordinary Diophantine exponent $\omega = \omega(\Theta)$ and the uniform Diophantine exponent $\hat{\omega} = \hat{\omega}(\Theta)$ defined as

$$\omega = \omega(\Theta) = \sup \left\{ \gamma : \liminf_{t \to +\infty} t^\gamma \psi_\Theta(t) < +\infty \right\},$$

$$\hat{\omega} = \hat{\omega}(\Theta) = \sup \left\{ \gamma : \limsup_{t \to +\infty} t^\gamma \psi_\Theta(t) < +\infty \right\}.$$

It is clear that

$$\frac{1}{n} \leqslant \hat{\omega} \leqslant 1$$

and

$$\omega \geqslant \hat{\omega}. \tag{1}$$

V. Jarník, W. Schmidt, N. Moshchevitin and D. Roy proved some results, concerning the relation $\frac{\omega}{\hat{\omega}}$ for n equals 2 and 3. In my talk I will tell you about some results from the joint paper with N.Moshchevitin, which is the first work, where the case $n = 4$ is considered.