A BIJECTIVE PROOF OF SHAPIRO’S CATALAN CONVOLUTION

GÁBOR V. NAGY
Bolyai Institute, University of Szeged, Szeged, Hungary

Using generating functions, Shapiro proved the following elegant convolution formula involving Catalan numbers of even index:

\[ \sum_{k=0}^{n} C_{2k} C_{2n-2k} = 4^n C_n. \]

Stanley asked for a bijective proof of this identity. We present a bijective proof, based on a path-counting argument. This is joint work with Péter Hajnal.

This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP 4.2.4.A/2-11-1-2012-0001 ‘National Excellence Program’.