Exponent of irrationality of some numbers.

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For any real irrational number α the *exponent of irrationality* $\mu(\alpha)$ is defined as the supremum of the set of all numbers $\varkappa$ such that inequality

$$\left|\alpha - \frac{p}{q}\right| < q^{-\varkappa}$$

has infinitely many solutions in rational numbers $\frac{p}{q}$. We know that $\mu(e) = 2$ and $\mu(\alpha) = 2$ for any algebraic irrational number α, K. Roth, 1954. Unfortunately precise values of $\mu(\alpha)$ are known only for a small set of numbers. For some classical constants we know only upper bounds for the exponent of irrationality. The best up today bounds are

$\mu(\pi) \leq 7,606308\ldots$	V. Salikhov, 2008,
$\mu(\pi^2) = \mu(\zeta(2)) \leq 5,09541178\ldots,$	W. Zudilin, 2013,
$\mu(\zeta(3)) \leq 5,513890\ldots,$	J. Rhin and C. Viola, 2001,
$\mu(\ln 2) \leq 3,57455390\ldots,$	R. Marcovecchio, 2009.

All known upper bounds for the irrationality exponents are based on effective constructions of rational approximations to numbers under consideration. In the talk we will give a survey of such constractions and specially discuss results noted above. Additionally we discuss a construction of approximations to Catalan constant $G = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$. It is still insufficient to prove irrationality of G.