# Hypergraphs with forbidden edge-intersections: new results and applications to Euclidean Ramsey theory*

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In 1981 Frankl and Wilson proved the following, now classical, theorem.

**Theorem 1.** *Let $H = (V, E)$ be a $k$-uniform hypergraph on $n$ vertices. Assume that for any $F_1, F_2 \in E$ we have $|F_1 \cap F_2| \neq l$ and $k - l$ is a prime power, which we denote by $q$. Then two cases take place:*

1. *if $2l < k$, then*

$$|E| \leqslant \sum_{i=0}^{q-1} C_n^i;$$

2. *if $2l \geqslant k$, then for $d = k - 2q + 1 = 2l - k + 1$,*

$$|E| \leqslant \frac{C_n^d}{C_k^d} \left( \sum_{i=0}^{q-1} C_{n-d}^i \right).$$

A bit later — in 1987 — Frankl and Rödl proved another important result.

**Theorem 2.** *Let $H_1 = (V, E_1)$, $H_2 = (V, E_2)$ be two hypergraphs with a common vertex set $V$ of size $n$. Assume that for any $F_1 \in E_1$, $F_2 \in E_2$ we have $|F_1 \cap F_2| \neq l$. If for some $\eta \in \left(0, \frac{1}{4}\right)$ we have $\eta n \leqslant l \leqslant \left(\frac{1}{2} - \eta\right) n$, then there exists a constant $\varepsilon > 0$ (depending only on $\eta$) such that $|E_1| \cdot |E_2| \leqslant (4 - \varepsilon + o(1))^n$.*

On the one hand, in a joint work with Ponomarenko we found an unexpected and substantial improvement to the bound in the second case of Theorem 1. On the other hand, in a joint work with Harlamova, Samirov and Zvonarev we succeeded in applying the improved version of Frankl–Wilson's theorem to find very accurate bounds for the value of $\varepsilon$ in Frankl–Rödl's theorem. Finally, we combined both improved Theorem 1 and Theorem 2 to obtain a series of strong lower bounds for the chromatic numbers of spaces with forbidden simplices. For example, denote by $\chi_\Delta(\mathbb{R}^n)$ the *chromatic number of the Euclidean space with forbidden monochromatic equilateral triangles*, which is the minimum number of colors needed to color the space so that no three monochromatic points form a triangle with all side lengths 1. Then we can show that $\chi_\Delta(\mathbb{R}^n) \geqslant (1.052\ldots + o(1))^n$.

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