Oscillation theorems for irrationality measure functions

For a matrix

$$\Theta = \begin{pmatrix} \Theta_1^1 & \Theta_1^2 & \dots & \Theta_1^n \\ \Theta_2^1 & \Theta_2^2 & \dots & \Theta_2^n \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_m^1 & \Theta_m^2 & \dots & \Theta_m^n \end{pmatrix}$$

the following function is defined

$$\psi_\Theta(t) = \min_{\substack{x_i \in \mathbb{Z} \\ 1 \leqslant \max\limits_{1,\dots,n} |x_i| \leqslant t}} \max_{1 \leqslant j \leqslant m} \|\Theta_j^1 x_1 + \dots + \Theta_j^n x_n\|$$

where t is integer number and $\|\cdot\|$ stands for the distance to the nearest integer.

If n=1 and m=1 then Θ is a real number and

$$\psi_\alpha(t) = \min_{1 \leqslant q \leqslant t} \|q\alpha\|.$$

In paper [**?**] it is proved that for any two real numbers α, β, such as $\alpha \pm \beta \notin \mathbb{Z}$ the difference function

$$\psi_\alpha(t) - \psi_\beta(t)$$

changes its sign infinitely many times as $t \to \infty$.

Our main result is the following theorem.

Theorem. Let $n = 1$, $m \in \mathbb{Z}$ or $n = 2$, $m = 1$ then for almost all matrix the *difference function*

$$\psi_\Theta(t) - \psi_{\Theta'}(t)$$

changes its sign infinitely many time as $t \to +\infty$.