Oscillation theorems for irrationality measure functions

For a matrix
\[
\Theta = \begin{pmatrix}
\Theta_1^1 & \Theta_1^2 & \ldots & \Theta_1^n \\
\Theta_2^1 & \Theta_2^2 & \ldots & \Theta_2^n \\
\vdots & \vdots & \ddots & \vdots \\
\Theta_m^1 & \Theta_m^2 & \ldots & \Theta_m^n \\
\end{pmatrix}
\]
the following function is defined
\[
\psi_\Theta(t) = \min_{x \in \mathbb{Z}} \max_{1 \leq j \leq m} \|\Theta_j^1 x_1 + \ldots + \Theta_j^n x_n\| \\
\text{where } t \text{ is integer number and } || \cdot || \text{ stands for the distance to the nearest integer.}
\]

If n=1 and m=1 then Θ is a real number and
\[
\psi_\alpha(t) = \min_{1 \leq q \leq t} \|q\alpha\|.
\]

In paper [?] it is proved that for any two real numbers \(\alpha, \beta\), such as \(\alpha \pm \beta \not\in \mathbb{Z}\) the difference function
\[
\psi_\alpha(t) - \psi_\beta(t)
\]
changes its sign infinitely many times as \(t \to \infty\).

Our main result is the following theorem.

**Theorem.** Let \(n = 1, m \in \mathbb{Z}\) or \(n = 2, m = 1\) then for almost all matrix
the difference function
\[
\psi_\Theta(t) - \psi_\Theta'(t)
\]
changes its sign infinitely many time as \(t \to +\infty\).