Palanga Conference in Combinatorics and Number Theory

Palanga, Lithuania
September 01 — September 07, 2013

PROGRAM
AND
ABSTRACT BOOK

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Conference Program

Monday, September 02

9:00–9:40	P. Frankl "Extremal problems for finite sets"
9:50–10:30	G. Katona "Towards a structured Baranyai theorem"

Coffee

11:00–11:40	K. Matsumoto "A numerical study on the behaviour of the Euler double zeta-function"
11:50–12:20	H. Mishou "A new method to prove joint universality for zeta functions"

Lunch

14:10–14:40	V. Lev "$a + b = 2c$"
14:50–15:20	R. Pinchasi "On the union of arithmetic progressions"
15:30–16:00	R. Okazaki "Totally imaginary quartic Thue equation with three solutions"

Coffee

16:30–17:00	A. Dubickas "Density of some geometric sequences modulo 1"
17:10–17:30	V. Zhuravleva "On the distribution of powers of some Pisot numbers"
17:40–18:00	R. Macaitienė "On the mixed joint universality"
18:10–18:30	D. Siaučiūnas "Universality of composite functions of periodic zeta-functions"
18:40–19:10	I. Shkredov "The eigenvalues method in Combinatorial Number Theory"

Tuesday, September 03

9:00–9:40	D. Bilyk "Discrepancy theory and harmonic analysis"
9:50–10:30	R. Nair "Ergodic Methods for Continued Fractions"

Coffee

11:00–11:40	S. Kristensen "Fourier transforms, normality and Littlewood type problems"
11:50–12:30	G. Alkauskas "Transfer operator for the Gauss' continued fraction map. Structure of the eigenvalues and trace formulas"

Lunch

14:10–14:50	G. Kabatyanskii "On a Discrete Compressed Sensing Problem"
15:00–15:30	L. Tóth "On the number of subgroups of finite Abelian groups of rank two and three"
15:40–16:10	M. Krzywkowski "On trees with double domination number equal to 2-domination number plus one"

Coffee

16:30–17:00	A. Voronenko "On one property of almost all Boolean functions"
17:10–17:40	A. Raigorodskii "Forbidding distances on $(0,1)$- and $(-1,0,1)$-spaces: new results and their applications in combinatorial geometry"
17:50–18:30	A. Sapozhenko "On the number of sets free from solutions of linear equations in abelian groups"
18:40–19:10	M. Zhukovskii "Critical points in zero-one laws for $G(n,p)$"

Wednesday, September 04

Excursion to Thomas Mann's House in Nida and Conference Party

Thursday, September 05

9:00–9:40	A. Ivić "Riemann zeta-function and divisor problem in short intervals"
9:50–10:30	J. Steuding "One Hundred Years Uniform Distribution Modulo One and Recent Applications to Riemann's Zeta-Function"

Coffee

11:00–11:30	M. Korolev "On L_p-norms of certain trigonometric polynomials"
11:40–12:10	I. Rezvyakova "Zeros of the Epstein zeta function on the critical line"
12:20–12:40	T. Onozuka "The multiple Dirichlet product and the multiple Dirichlet series"

Lunch

14:30–14:50	N. Christ "A-point-distribution of the Riemann zeta-function near the critical line"
15:00–15:20	T. Srichan "Sampling the Lindelöf Hypothesis for Hurwitz zeta-functions by a Cauchy random walk"
15:30–16:00	J. Kalpokas "Value distribution of the Riemann zeta function on the critical line"

Coffee

16:30–17:00	T. Nakamura "Zeta distributions and quasi-infinite divisibility"
17:10–17:40	M. Suzuki "An inverse problem for a class of canonical systems and its applications"
17:50–18:20	P. Šarka "Least common multiple of random sets"
18:30–19:00	V. Zacharovas "Limit distribution of the coefficients of polynomials with only unit roots"

Friday, September 06

9:00–9:30	L. Ostroumova "Recency-based preferential attachment models"
9:40–10:10	V. Dolnikov "On the chromatic number of a hypergraph with (p, q)-property"
10:20–10:50	V. Koshelev "On the chromatic variant of the Erdős–Szekeres problems with limited numbers of interior points"

Coffee

11:20–11:50	R. Garunkštis "On the Speiser equivalent for the Riemann hypothesis"
12:00–12:30	A. Laurinčikas "Some remarks on universality of zeta-functions"

Lunch

14:30–14:50	D. Kruchinin "On the Central Coefficients of Triangles"
15:00–15:20	V. Parusnikov "The Lagrange theorem for continued fractions of inhomogeneous linear forms"
15:30–16:00	O. German "A family of transference theorems"
16:10–16:30	N. Moshchevitin "On a question by N. Chevallier"

Coffee

Giedrius Alkauskas (Lithuania, Vilnius)

Transfer operator for the Gauss' continued fraction map. Structure of the eigenvalues and trace formulas

Let $\mathcal{L}$ be the transfer operator associated with the Gauss' continued fraction map, known also as the *Gauss-Kuzmin-Wirsing operator*, acting on the Banach space. In this talk we describe a work where we prove an asymptotic formula for the eigenvalues of $\mathcal{L}$, denoted by λ_n, $n \in \mathbb{N}$. This settles, in a stronger form, the conjectures of D. Mayer and G. Roepstorff (1988), A.J. MacLeod (1992), Ph. Flajolet and B. Vallée (1995), also supported by several other authors. Further, we find an exact series for the eigenvalues, which also gives the canonical decomposition of trace formulas due to D. Mayer (1976) and K.I. Babenko (1978). This crystallizes the contribution of each individual eigenvalue in the trace formulas.

Theorem (Arithmetic and decomposition of trace formulas)**.** *There exist functions $W_v(\mathbf{X})$, $v \geq 0$, defined by $W_0(\mathbf{X}) = 1$, $W_1(\mathbf{X}) = \frac{5}{4} \cdot \phi^{-2\mathbf{X}} P^{(0,1)}_{\mathbf{X}-1}(3/2)$ (ϕ is the golden section, $P^{(\alpha,\beta)}_n$ are Jacobi polynomials), and then by a certain explicit recurrence, such that*

$$(-1)^{n+1}\lambda_n = \phi^{-2n}\sum_{\ell=0}^{\infty} W_\ell(n), \quad |W_\ell(n)| < \frac{C}{\ell^2\sqrt{n}} \text{ for } \ell \geq 1,$$

for an absolute constant C. This decomposition is compatible and gives the decomposition of trace formulas (due to D. Mayer) for the powers of $\mathcal{L}$: for the first and second powers, we have, respectively,

$$\begin{aligned}\sum_{n=1}^{\infty}(-1)^{n+1}\phi^{-2n}W_{\ell-1}(n) &= \frac{1}{\xi_\ell^{-2}+1}, \quad \ell \geq 1,\\ \sum_{i+j=\ell}\sum_{n=1}^{\infty}\phi^{-4n}W_{i-1}(n)W_{j-1}(n) &= \sum_{i+j=\ell}\frac{1}{(\xi_{i,j}\xi_{j,i})^{-2}-1}, \quad \ell \geq 2.\end{aligned}$$

Analogously for higher powers of $\mathcal{L}$. Here ξ_ℓ and $\xi_{i,j}$ are quadratic irrationals: $\xi_\ell = [0, \overline{\ell}]$, $\xi_{i,j} = [0, \overline{i, j}]$.

Dmirtiy Bilyk (Minneapolis, USA)

Discrepancy theory and harmonic analysis

Discrepancy theory, a field pioneered by such giants as H. Weyl and K. Roth, studies different ways of approximating continuous objects by discrete ones and quantifies the errors that inevitably arise in such approximations.

The discrepancy of a set $\mathcal{P}_N \subset [0,1]^d$ consisting of N points with respect to a geometric family $\mathcal{A}$ is defined as

$$D_N = \sup_{A \in \mathcal{A}} \big| \# \mathcal{P}_N \cap A - N \cdot vol(A) \big|,$$

i.e. the maximal difference between the actual and expected number of points of $\mathcal{P}_N$ that fall into sets of $\mathcal{A}$. Appropriate L^2 averages may also be used in place of the supremum.

It is well known that when $\mathcal{A}$ consists of rectangles rotated in arbitrary directions, then the discrepancy behaves as a fractional power of N, while for axis-parallel rectangles the discrepancy is logarithmic. The exact power of the logarithm, however, is still a mystery in dimensions higher than two, even on the level of conjectures. The recent estimate of the speaker, Lacey, and Vagharshakyan

$$D_N \gtrsim (\log N)^{\frac{d-1}{2}+\varepsilon}$$

yields the first higher-dimensional improvement of Roth's 1954 L^2 bound. The problem turns out to be related to small deviation probabilities for multiparameter Gaussian processes, entropy bounds for mixed Sobolev spaces, and certain inequalities of product harmonic analysis.

On the other hand, in joint work with Ma, Pipher, and Spencer, we attempted to look at the transition between polynomial and logarithmic estimates by studying discrepancy with respect to families of rectangles rotated in specific sets of directions. We construct rotated lattices which have low discrepancy for a given set of rotations depending on its covering properties. The obtain these bounds, we prove certain simultaneous Diophantine approximation results which rely on methods of Davenport and Peres–Schlag.

Ilya I. Bogdanov, Vladimir L. Dolnikov (Yaroslavl, Russia)

On the chromatic number of a hypergraph with (p,q)-property

In this report we talk about the chromatic number of a uniform hypergraph with (p,q)-property. Let us recall necessary definitions.

A *hypergraph* is a pair $G=(V,E)$, where V is a set, and $E\subseteq 2^V$. The elements of V are *vertices* of G, and the elements of E are its *(hyper)edges*. A hypergraph $G=(V,E)$ is r*-homogeneous* (or simply an r*-graph*) if all its edges have cardinality r. Thus, 2-graphs are just usual graphs.

A set $U\subset V$ of a hypergraph $G=(V,E)$ is *independent*, if it contains no edges. A *coloring* of a hypergraph is a partition of the set V into several disjoint independent parts: $V=V_1\sqcup V_2\sqcup\cdots\sqcup V_t$. The *chromatic number* $\chi(G)$ of a hypergraph G is the minimal number of colors in its coloring.

The notion of (p,q)-property was initially introduced by Hadwiger and Debrunner for the families of convex subsets in $\mathbb{R}^n$ in connection with the investigation of Helly–Gallai properties of these families. For such a family $\mathcal{F}$, it is often convenient to consider its a $(n+1)$–graph with $\mathcal{F}$ as the set of vertices; its hyperedges are exactly those $(n+1)$–element subfamilies of $\mathcal{F}$ which have nonempty intersections. In view of this, it is convenient to introduce the following definition.

Let $G=(V,E)$ be a hypergraph, and p, q be integer numbers with $p\geq q\geq 1$. We say that G satisfies the (p,q)*-property* if $|V|\geq p$, and every subset $V'\subseteq V$ with $|V|\geq p$ contains an independent subset with q elements.

The main result of ths talk are the following theorem.

Theorem. Suppose r, p, q, $p\geq q\geq r$ are positive integers, and $d=p-q$.

If $p<r(d+1)$, then for every N there exists an r-graph G satisfying the (p,q)-property such that $\chi(G)>N$.

If $p\geq r(d+1)$, then for every r-graph G satisfying the (p,q)-property we have

$$\left\lceil\frac{d}{r-1}\right\rceil+1\leq\chi(G)\leq\left\lceil\frac{d}{r-1}\right\rceil+2.$$

Thomas Christ (Würzburg, Germany)

A-point-distribution of the Riemann zeta-function near the critical line

For a given $a \in \mathbb{C}$, we refer to the roots of the equation $\zeta(s) = a$ as a-points of the Riemann zeta-function. In 1913, Landau established a Riemann-von Mangoldt type formula for the number of a-points in the upper half-plane. Additionally, he observed that the a-points seem to cluster around the critical line. Levinson proved in 1975 that almost all a-points (in the sense of density) can be found in the region defined by

$$\sigma + it \in \mathbb{C} \quad : \quad \frac{1}{2} - \frac{(\log\log t)^2}{\log t} < \sigma < \frac{1}{2} + \frac{(\log\log t)^2}{\log t}, \qquad t \geq 2.$$

Refining Levionson's method with results of Selberg and Tsang, we can narrow the region above such that there are still almost all a-points lying inside.

Moreover, by using certain arguments from the theory of normal families, we try to figure out how fast a positive function $\lambda(t)$ can tend to 0, as $t \to \infty$, such that there are still infinitely many a-points lying inside the region defined by

$$\sigma + it \in \mathbb{C} \quad : \quad \frac{1}{2} + \lambda(t) < \sigma < \frac{1}{2} + \lambda(t), \qquad t \geq 2.$$

Our methods are not limited to the Riemann zeta-function but can be transfered easily to appropriate other L-functions.

Artūras Dubickas (Artūras Dubickasilnius, Lithuania)

Density of some geometric sequences modulo 1

Let $\{x\}$ be the fractional part of $x \in \mathbb{R}$. Recently, Cilleruelo, Kumchev, Luca, Rué and Shparlinski showed that for each integer $a \geq 2$ the sequence $\{a^n/n\}_{n=1}^{\infty}$ is everywhere dense in the interval $[0, 1]$.

We prove the following theorem:

Theorem. *If α is a Pisot number or a Salem number and $Q(z)$ is a non-constant polynomial with integer coefficients then the sequence $\{Q(\alpha^n)/n\}_{n=1}^{\infty}$ is everywhere dense in $[0, 1]$. Furthermore, for any $c > 0$ and any sufficiently large integer N every interval $J \subseteq [0, 1]$ of length $|J| \geq cN^{-0.475}$ contains at least one element of this sequence with the index n in the range $1 \leq n \leq N$.*

Péter Frankl (Tokyo, Japan)

Extremal problems for finite sets

Let X be a finite set of n elements. A family $\mathcal{F}$ is a subset of 2^X , the power set of X. The generic question is as follows. Supposing that $\mathcal{F}$ satisfies certain conditions determine or estimate the maximum of the size of $\mathcal{F}$.

The simplest conditions are:

- no member of $\mathcal{F}$ contains another member (Sprener Theorem),
- any two members of $\mathcal{F}$ have non-empty intersection (Erdos-Ko-Rado Theorem),
- any two members of $\mathcal{F}$ intersect in at least t elements (Katona Theorem).

All the above are by now classical results, 40 years ago the author proposed the more general condition: any r members of $\mathcal{F}$ intersect in at least t elements (r and t are fixed positive integers).

The general answer is still unknown.

The lecture will review these and related problems.

Ramūnas Garunkštis (Vilnius, Lithuania)

On the Speiser equivalent for the Riemann hypothesis

Speiser showed that the Riemann hypothesis is equivalent to the absence of non-trivial zeros of the derivative of the Riemann zeta-function left of the critical line. We investigate the relationship between the non-trivial zeros of the extended Selberg class functions and of their derivatives left of the critical line. Every element of this class satisfies a functional equation of Riemann type, but it contains zeta-functions for which the Riemann hypothesis is not true. As an example, we study the relationship between the trajectories of zeros of a certain linear combination of Dirichlet L-functions and of its derivative computationally. In addition, we examine Speiser type equivalent for Dirichlet L-functions with imprimitive characters for which the Riemann hypothesis is not true and which do not satisfy a Riemann type functional equation. This is a joint work with Raivydas Šimėnas.

Oleg N. German, Konstantin G. Evdokimov
(Moscow, Russia)

A family of transference theorems[1]

Our talk is devoted to the relation between successive minima of pseudocompound parallelepipeds, i.e. parallelepipeds M, M^* of the form

$$M = \{\, \mathbf{z} \in \mathbb{R}^d \ :\ |\langle \boldsymbol{\ell}_i, \mathbf{z} \rangle| \leqslant \lambda_i, \qquad i = 1, \ldots, d \,\},$$

$$M^* = \{\, \mathbf{z} \in \mathbb{R}^d \ :\ |\langle \boldsymbol{\ell}_i^*, \mathbf{z} \rangle| \leqslant \lambda/\lambda_i\,, \ \ i = 1, \ldots, d \,\},$$

where $\boldsymbol{\ell}_1, \ldots, \boldsymbol{\ell}_d$ and $\boldsymbol{\ell}_1^*, \ldots, \boldsymbol{\ell}_d^*$ are dual bases of $\mathbb{R}^d$, $\lambda = (\lambda_1 \cdot \ldots \cdot \lambda_d)^{\frac{1}{d-1}}$, and $\langle \,\cdot\,, \,\cdot\, \rangle$ denotes the inner product.

The classical result in this area belongs to Kurt Mahler. He showed in 1937 that the first successive minima of M and M^* w.r.t. $\mathbb{Z}^d$ satisfy the relation

$$\mu_1(M) \leqslant 1 \quad \Longrightarrow \quad \mu_1(M^*) \leqslant d-1.$$

We consider the second minimum of M^* and prove the following

Theorem 1. *We have*

$$\mu_1(M) \leqslant 1 \quad \Longrightarrow \quad \left[\begin{array}{l} \mu_1(M^*) \leqslant 1 \\ \mu_2(M^*) \leqslant c_d \end{array} \right.,$$

where c_d is the positive root of the polynomial $t^{2(d-1)} - 2(d-1)t - 1$.

Moreover, for $d = 3$ we have

$$\mu_1(M) \leqslant 1 \quad \Longrightarrow \quad \left[\begin{array}{l} \mu_1(M^*) \leqslant 1 \\ \mu_2(M^*) \leqslant \frac{\sqrt{3}+\sqrt{7}}{2\sqrt{3}} \end{array} \right..$$

[1]This research was supported by RFBR grant N° 12–01–31106 and by the grant of the President of Russian Federation N° MK–5016.2012.1

Since $c_d < d-1$ and $c_d \to 1$ as $d \to \infty$, Theorem 1 improves upon Mahler's theorem. Moreover, Theorem 1 is actually a corollary to a stronger statement which describes a certain $(d-1)$–parametric family of parallelepipeds, each containing nonzero integer points, provided $\mu_1(M) \leqslant 1$.

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Aleksandar Ivić (Belgrad, Serbia)

Riemann zeta-function and divisor problem in short intervals

Let the error term in the classical Dirichlet divisor problem be

$$\Delta(x) \ := \ \sum_{n\le x} d(n) - x(\log x + 2\gamma - 1),$$

and in the mean square formula for the zeta-function

$$E(T) \ := \ \int_0^T |\zeta(\frac{1}{2}+it)|^2 \,\mathrm{d}t - T\left(\log(\frac{T}{2\pi}) + 2\gamma - 1\right).$$

Here $d(n)$ is the number of divisors of n, $\zeta(s)$ is the Riemann zeta-function, and γ is Euler's constant. F.V. Atkinson's (1949) classical explicit formula for $E(T)$ provides the analogy between $\Delta(x)$ and $E(T)$. A better analogy is obtained with the use of the function

$$\Delta^*(x) := -\Delta(x) + 2\Delta(2x) - \frac{1}{2}\Delta(4x)$$

$$= \frac{1}{2}\sum_{n\le 4x}(-1)^n d(n) - x(\log x + 2\gamma - 1),$$

and then with the function

$$E^*(t) \ := \ E(t) - 2\pi\Delta^*\bigl(\frac{t}{2\pi}\bigr).$$

Several results involving power moments of $|\zeta(\frac{1}{2}+it)|$, which depend on various bounds for power moments of $E^*(T)$ (in short intervals) are presented in this talk.

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Grigory Kabatiansky[1] (Moscow, Russia)

On a Discrete Compressed Sensing Problem

The Compressed Sensing Problem [1],[2] is formulated as finding a t-sparse solution vector $x \in \mathbb{R}^n$ of the following equation

$$s = Hx^T + e,$$

where H is an $r \times n$ matrix, and vector $s = (s_1, \ldots, s_r)$ we shall call *syndrom*. Formulated in such way the problem resembles, as it has been noticed many times, the main problem of coding theory. Usual for compressed sensing assumption that vector e has relatively small Euclidean norm has not much sense in discrete, especially in binary, case. Therefore we replace it on assumption that the Hamming weight $wt(e) \leq l$. More formally

Definition 1. *A q -ary $r \times n$ matrix H called a (t, l)-compressed sensing (CS) matrix iff $d(Hx^T, Hy^T) \geq 2l + 1$ for any two distinct vectors $x, y \in \mathbb{F}_q^n$ such that $wt(x) \leq t$ and $wt(y) \leq t$.*

Remark. In particular case $t = 1,\ l = 1$ we have got nonadaptive (or deterministic) version of famous Ulam's problem on searching with lie, see [3], and for $t = 1$ and l arbitrary — a nonadaptive version of Ulam's problem on searching with lies.

Let $r_q(n, t; l)$ be the minimal redundancy r taken over all q -ary $r \times n$ (t, l)-CS matrices. Denote by $A_q(n, 2t + 1)$ the maximal possible cardinality of q-ary code of length n correcting t errors and by $V_q(n, d) = \sum_{i=0}^{i=d} C_n^i (q-1)^i$ the volume of radius d ball in q-ary n-dimensional Hamming space. We prove the following analogs of Hamming and Gilbert-Varshamov bounds.

Theorem 1. *(Hamming bound). For any q -ary (t, l)-CS $r \times n$-matrix*

$$A_q(r, 2l + 1) \geq V_q(n, t).$$

[1]Grigory Kabatiansky, Institute for Information Transmission Problems (IITP RAS), Moscow 127994, Russia (e-mail: kaba@iitp.ru). The work of G.Kabatiansky was supported by RFBR grants 13-07-00978 and 12-01-00905.

On the other hand, there is the following analog of the Gilbert-Varshamov bound.

Theorem 2. *(G-V bound).*

$$r_q(n,t;l) \leq \log_q V_q(n,2t) + \log_q V_q(r,2l).$$

References

[1] D.L. Donoho, "Compressed sensing", IEEE Transactions on Information Theory, v. 52 (4), pp. 1289-1306, 2006

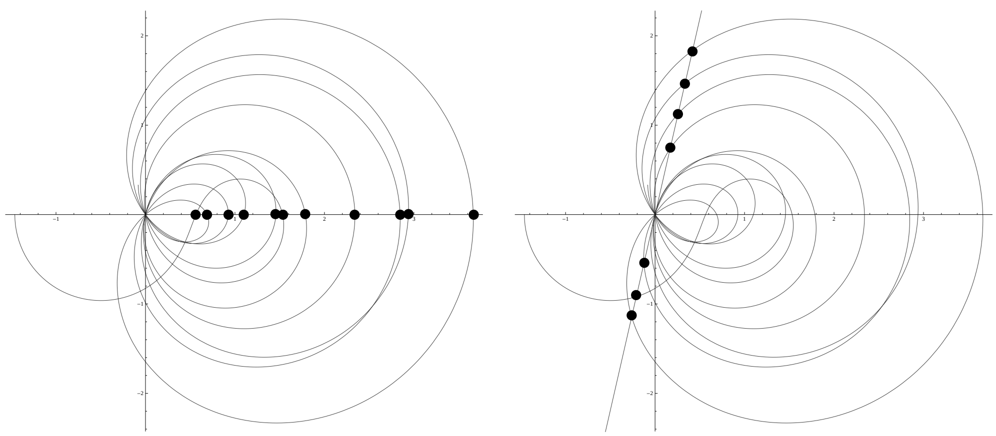
[2] E.J. Candes, T. Tao, " Near-Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Transactions on Information Theory, v. 52 (12), pp. 5406 - 5425, 2006

[3] A. Pelc, "Solution of Ulam's Problem on Searching with a Lie", Journal of Comb.Theory, Ser.A 44, pp129-140, 1987.

Justas Kalpokas (Vilnius, Lithuania)

Value distribution of the Riemann zeta function on the critical line

In [3] Kalpokas and Steuding introduced a method to generalise Gram's points (at Gram's points the Riemann zeta function obtains real values on the critical). The method helps to investigate the values of the Riemann zeta function that appears from the intersection points of a straight line and the curve of the Riemann zeta function (see Figure 1).



(a) (b)

Figure 1: Curve $t \mapsto \zeta(\frac{1}{2}+it)$, where t varies from 0 to 50: (a) black thick dots are values of $\zeta(1/2+ig_n)$ at Gram's points $1/2+ig_n = 1/2+it_n(0)$ and (b) black thick dots are values of $\zeta(1/2+it_n(\frac{3}{7}\pi))$ at generalised Gram's point $1/2+it_n(\frac{3}{7}\pi)$.

Recall that $e^{-i\phi}\zeta(\frac{1}{2}+it_n(\phi))$ is real, here $\phi \in [0,\pi)$ and indicates the angle of the straight line (crossing the origin) with the Real line. Hence, we may write $t_n^+(\phi)$ in place of $t_n(\phi)$ if $e^{-i\phi}\zeta(\frac{1}{2}+it_n(\phi)) \geqslant 0$ and $t_n^-(\phi)$ if $e^{-i\phi}\zeta(\frac{1}{2}+it_n(\phi)) < 0$ (see Figure 1 (b)). In [2] Kalpokas, Korolev and Steuding proved the following result.

Theorem 1. *For any $\phi \in [0,\pi)$, there are arbitrary large positive and nega-*

tive values of $e^{-i\phi}\zeta(\frac{1}{2}+it_n(\phi))$*. More precisely,*

$$\max_{0<t_n^{\pm}(\phi)\leqslant T}\left|\zeta(\tfrac{1}{2}+it_n^{\pm}(\phi))\right| \gg (\log T)^{\frac{5}{4}}.$$

If the Riemann hypothesis is assumed then for any arbitrary small $\delta > 0$ *we have*

$$\max_{0<t_n^{\pm}(\phi)\leqslant T}\left|\zeta(\tfrac{1}{2}+it_n^{\pm}(\phi))\right| \gg (\log T)^{\frac{3}{2}-\delta}.$$

The separate case of the theorem can be stated as "the Riemann zeta function has infinitely many negative values on the critical line and those values are unbounded".

To prove the second part of the Theorem 1 (with the Riemann Hypothesis assumption) we used a result from [1].

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Gyula O.H. Katona (Budapest, Hungary)

Towards a structured Baranyai theorem

Theorem 1 (Baranyai [1] [2]). *If k divides n then there are $\binom{n-1}{k-1}$ partitions of the n-element set into k-element subsets in such a way that every k-element subset occurs in exactly one of these partitions.*

We will show some conjectures generalizing this theorem. For instance what may one say when k does not divide n.

However nothing is known (and not even conjectured) about the pairwise relation of the partitions. For instance, what are the sizes of the intersections if two such partitions are taken? It seems to be difficult to obtain results of this sort. This is why we consider some weaker structures rather than partitions.

The objects considered here will be families of ℓ pairwise disjoint k-element sets. They are called (k,ℓ)-*partial partitions* or shortly (k,ℓ)-*papartitions*. We say that two papartitions are *not too close to each other* if there are no classes A_1, A_2 in one of them, B_1, B_2 in the other one satisfying

$$A_1 \cap B_1 > \frac{k}{2}, \text{ and } A_2 \cap B_2 > \frac{k}{2}.$$

Theorem 2 (Gyula O.H. Katona, Gyula Y. Katona). *Let k and ℓ be positive integers. If n is large then one can find*

$$\frac{\binom{n}{k}}{\ell}$$

(k,ℓ)-papartitions in such a way that no k-element set appears in two of them and they are not too close to each other.

The proof is based on a theorem which is a far generalization of Dirac's theorem on Hamiltonian cycles. It is a statement on two graphs on the same vertex set, and claims that if the minimum degree of the first graph is very large, while the maximum degree of the second one is small then the first graph can be decomposed into vertex-disjoint complete ℓ-vertex graphs in such a way that certain configurations between these complete graphs and the second graph are avoided.

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Maxim A. Korolev[1] (Moscow, Russia)

On L_p-norms of certain trigonometric polynomials

The following polynomials arise in different problems of number theory:

$$V_x(t) \,=\, \sum_{p\leqslant x}\frac{\sin{(t\log p)}}{\sqrt{p}}.$$

Here p runs through prime numbers, and the parameters t and $x = x(t)$ increase unboundedly. The main goal of the talk is to describe some new formulas for the moments of $V_x(t)$, i.e. for the integrals

$$I_a(T) \,=\, \int_0^T \bigl|V_x(t)\bigr|^{2a}dt, \qquad a>0.$$

In particular, the following results will be presented.

Theorem 1. Let $a = k \geqslant 1$ be an integer. Suppose that $x_0 \leqslant x \leqslant T^{1/(2k)}$. Then

$$I_k(T) \,=\, \frac{T}{2^{2k}}\sum_{n=0}^{k}\frac{(2k)!}{(k-n)!}\,\Phi_n\mathfrak{S}^{k-n} \,+\, \theta\, x^{2k},$$

where $|\theta| \leqslant 1$, $\mathfrak{S} = \sum\limits_{p\leqslant x} p^{-1}$ and the coefficients Φ_n are expressed as a polynomials in variables $\mathfrak{S}_r = \sum\limits_{p\leqslant x} p^{-r}$, $r = 2,3,\ldots,n$. In particular,

$$\Phi_0 \equiv 1, \quad \Phi_1 \equiv 0, \quad \Phi_2 = -\frac{1}{2^2}\,\mathfrak{S}_2, \quad \Phi_3 = \frac{1}{3^2}\,\mathfrak{S}_3,$$
$$\Phi_4 = -\,\frac{11}{2^6\cdot 3}\,\mathfrak{S}_4 \,+\, \frac{1}{2^5}\,\mathfrak{S}_2^2, \quad \Phi_5 \,=\, \frac{19}{2^3\cdot 3\cdot 5^2}\,\mathfrak{S}_5 \,-\, \frac{1}{2^2\cdot 3^2}\,\mathfrak{S}_2\mathfrak{S}_3.$$

[1]The author is supported by RFBR (grant No. 12-01-31165.)

Theorem 2. Let $x_0 \leqslant x \leqslant T^{0.0001}$, and let a be any non-integer under the conditions $0 < a < \frac{1}{6e}\sqrt{\frac{\log T}{\log x}}$. Then

$$I_a(T) \;=\; \frac{T}{\sqrt{\pi}}\,\Gamma(a+0.5)\biggl\{\,\sum_{n=0}^{+\infty}(-1)^n\,\frac{\Gamma(n-a)}{\Gamma(-a)}\,\Phi_n\mathfrak{S}^{a-n} \;+\; O(R_a)\biggr\},$$

where

$$R_a \;=\; \biggl(\frac{c}{\mathfrak{S}}\,\frac{\log x}{\log T}\;\max{(2a,1)}\biggr)^a, \quad c > 0.$$

The following applications of the above results to the function $S(t) = \pi^{-1}\arg\zeta\bigl(\frac{1}{2}+it\bigr)$ and to the Gram's law will also be discussed.

Theorem 3. Suppose that $T \geqslant T_0 > 0$ and let $L = \log\log T$. Then the formula

$$\int_0^T |S(t)|^{2a}dt \;=\; \frac{\Gamma(a+0.5)}{\pi^{2a+0.5}}\,TL^a\bigl(1 \;+\; O(r_a(T))\bigr)$$

holds true for any a, $0 < a \leqslant c_1\sqrt[3]{L}$, with

$$r_a(T) \;=\; \begin{cases} \bigl(L^{-1}\log L\bigr)^a & \text{for } \; 0 < a \leqslant 0.5, \\ aL^{-0.5}\bigl(\sqrt{a}+\sqrt{\log L}\bigr), & \text{for } \; a > 0.5. \end{cases}$$

Moreover, the term $\sqrt{\log L}$ can be omitted if $a = k$ is integer.

Let γ_n be the positive ordinates of zeros of $\zeta(s)$ counted with multiplicities ant let t_n be Gram points. For given γ_n there exists a unique $m = m(n)$ such that $t_{m-1} < \gamma_n \leqslant t_m$. Following to Selberg, we define $\Delta_n = m - n$.

Theorem 4. Suppose that $N \geqslant N_0 > 0$ and let let $L = \log\log N$. Then the formula

$$\sum_{n\leqslant N}|\Delta_n|^{2a} \;=\; \frac{\Gamma(a+0.5)}{\pi^{2a+0.5}}\,NL^a\bigl(1 \;+\; O(\rho_a)\bigr)$$

holds true for any a, $0 < a \leqslant c_1\sqrt[3]{L}$, with $\rho_a = r_a(N)$ defined above.

The last part of the talk deals with possible extensions of these results to more wide class of polynomials of the type

$$\sum_{p \leqslant x} a_p \, f(t \log p),$$

where a_p is any real sequence satisfying some natural conditions and $f(u)$ denotes any function e^{iu}, $\sin u$, $\cos u$.

Vitalii A. Koshelev[1] (Moscow, Russia)

On the chromatic variant of the Erdős–Szekeres problems with limited numbers of interior points[2]

In 1935 P. Erdős and G. Szekeres proposed the following problem, which is now classical and one of the most famous in combinatorial geometry: *determine, for any $n \geq 3$, the smallest number $g(n)$ such that in every set of $g(n)$ points in $\mathbb{R}^2$ in general position, one can find n vertices of a convex n-gon.*

Recall that a set of points in the plane is in *general position* if any three of its elements do not lie in a straight line.

Many generalizations and related problems have been proposed and many researchers studied them. In particular, *empty* convex polygons were considered. Moreover, numbers of interior points were counted. Also, special point sets were investigated as well as colored sets, etc.

In this talk, we deal with the following important generalization: *determine, for any $n_1 \geq 3$, $n_2 \geq 3$, $k_1 \geq 0$, $k_2 \geq 0$, the minimum number $h(n_1, k_1; n_2, k_2)$ (the minimum number $h_{nc}(\dots)$) such that in any two-colored set $\mathcal{X}$ of at least $h(n_1, k_1; n_2, k_2)$ (of at least $h_{nc}(\dots)$) points in general position in $\mathbb{R}^2$, there exists either a subset of cardinality n_1 whose elements form the set of vertices of a convex (of a non-convex) polygon C_1 of color 1 with the property $|(C_1 \setminus \partial C_1) \cap \mathcal{X}| \leq k_1$, or a subset of cardinality n_2 whose elements form the set of vertices of a convex (of a non-convex) polygon C_2 of color 2 with the property $|(C_2 \setminus \partial C_2) \cap \mathcal{X}| \leq k_2$.*

Note that this problem may be proposed for another number of colors. Our Theorems 1 – 3 relate to *the one-color* case of the problem (so the number of arguments of the quantity h becomes equal to 2).

Theorem 1. *If $n \geq 7$, then for even and odd values of n, respectively, the following quantities do not exist: $h\left(n, C_{n-7}^{(n-7)/2} - 1\right), h\left(n, 2C_{n-8}^{(n-8)/2} - 1\right)$.*

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[2]This work was supported by the grants of RFBR N 12-01-00683 and of the President of the Russian Federation N MD-6277.2013.1.

Theorem 2. *If $n \geq 6$, then* $h\left(n, \binom{(n-3)}{\lceil (n-3)/2 \rceil} - \lceil \frac{n}{2} \rceil\right) > 2^{n-2} + 1$.

Theorem 3. *The exact equalities hold:* $h(6, \geq 2) = 17$, $h(6, 1) = 18$.

Note that Theorem 3 gives the exact values of some of our quantities, which is important. It has a computer-based proof.

The convex version of the problem under consideration was proposed in 2003 by Devillers, Hurtado, Károlyi, and Seara. They proved that $h(3, 0; 3, 0) = 9$ and that the quantities $h(5, 0; 3, 0)$ and $h(3, 0; 3, 0; 3, 0)$ (here we have three-colored sets) do not exist. Therefore, the values $h(\geq 5, 0; \geq 3, 0)$ and $h(\geq 3, 0; \geq 3, 0; \geq 3, 0)$ do not exist too. The question about the existence and the exact value of $h(4, 0; 4, 0)$ is open.

It was known that $h(4, 0; 4, 0) \geq 36$. We present a new lower bound (and the corresponding point set) for this value: $h(4, 0; 4, 0) \geq 46$. It is easy to see that $h(3, 0; 4, 0) \leq h(6, 0)$ and, consequently, this value exists. A lower bound for it is $h(3, 0; 4, 0) \geq 22$.

Aichholzer, Hackl, Huemer, Hurtado, and Vogtenhuber studied the non-convex version of the problem. They proved an upper bound for $h_{nc}(4, 0; 4, 0)$: $h_{nc}(4, 0; 4, 0) \leq 2520$.

It was known that $h_{nc}(4, 0; 4, 0) \geq 18$. We present a new lower bound $h_{nc}(4, 0; 4, 0) \geq 20$.

Moreover, we find many exact values for $h(n_1, k_1; n_2, k_2)$ with $n_1, n_2 \leq 4$ and various k_1, k_2. For example, $h_{nc}(4, 1; 4, 1) = 11$, $h_{nc}(4, 0; 4, 1) = 15$, $h_{nc}(4, 0; 3, 0) = 14$, and $h(4, 0; 3, 1) = h(4, 1; 3, 0) = 14$.

Simon Kristensen (Aarhus, Denmark)

Fourier transforms, normality and Littlewood type problems

We explore some connections between Fourier transforms of measures, the almost sure normality of numbers and some results on problems resembling the Littlewood conjecture. In particular, we give some extensions of recent results by Haynes, Jensen and the speaker, and discuss connections with an inhomogeneous Littlewood conjecture of Cassels, which was recently resolved by Shapira. We will also discuss the mixed Littlewood conjecture of de Mathan and Teulie.

Dmitry V. Kruchinin (Tomsk, Russia)

On the Central Coefficients of Triangles

In this paper we study a problem of obtaining a generating function for central coefficients of a triangle $T(n,k)$, which is given by the expression $[xH(k)]^k = \sum_{n\geq k} T(n,k)x^n$. We present the method of obtaining a generating function for central coefficients of the given triangle (Theorem 1), which was introduced in [1]. The method is based on the Lagrange inversion formula and the notion of the composita.

First of all we consider the notion of the *composita* of a given ordinary generating function $G(x) = \sum_{n>0} g(n)x^n$, which was introduced in [2].

The composita is the coefficients of the powers of an ordinary generating function

$$F^{\Delta}(n,k) = [x^n]F(x)^k.$$

Lemma 1 (The Lagrange inversion formula)**.** *Suppose $H(x) = \sum_{n\geq 0} h(n)x^n$ with $h(0) \neq 0$, and let $A(x)$ be defined by*

$$A(x) = xH(A(x)).$$

Then

$$n[x^n]A(x)^k = k[x^{n-k}]H(x)^n,$$

where $[x^n]A(x)^k$ is the coefficient of x^n in $A(x)^k$ and $[x^{n-k}]H(x)^n$ is the coefficient of x^{n-k} in $H(x)^n$.

By using the Lagrange inversion formula, we give the main theorem.

Theorem 1. *Suppose $H(x) = \sum_{n\geq 0} h(n)x^n$ is a generating function, where $h(0) \neq 0$, and $G(x) = xH(x)$ with $[G(x)]^k = \sum_{n\geqslant k} G^{\Delta}(n,k)x^n$, and $A(x) = \sum_{n>0} a(n)x^n$ is the generating function, which is obtained from the functional equation $A(x) = xH(A(x))$. Then the generating function $F(x)$ for the central coefficients of the triangle $G^{\Delta}(n,k)$ is equal to the first derivative of the function $A(x)$:*

$$F(x) = A'(x) = \sum_{n\geqslant 1} G^{\Delta}(2n-1,n)x^{n-1}.$$

Also we give some examples of application of the method.

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Marcin Krzywkowski (Gdansk, Poland)

On trees with double domination number equal to 2-domination number plus one

A vertex of a graph is said to dominate itself and all of its neighbors. A subset $D \subseteq V(G)$ is a 2-dominating set of G if every vertex of $V(G) \setminus D$ is dominated by at least two vertices of D, while it is a double dominating set of G if every vertex of G is dominated by at least two vertices of D. The 2-domination (double domination, respectively) number of a graph G is the minimum cardinality of a 2-dominating (double dominating, respectively) set of G. We characterize all trees with the double domination number equal to the 2-domination number plus one. The result has recently appeared in the Houston Journal of Mathematics.

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Antanas Laurinčikas (Vilnius, Lithuania)

Some remarks on universality of zeta-functions

Let $s=\sigma+it$, $D=\left\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\right\}$, $\mathcal{K}$ be the class of compact subsets of the strip D with connected complements, and $H_0(K)$, $K\in\mathcal{K}$, be the class of continuous non-vanishing functions on K which are analytic in the interior of K. Then the universality of the Riemann zeta-function $\zeta(s)$ is the following assertion. Suppose that $K\in\mathcal{K}$ and $f(s)\in H_0(K)$. Then, for every $\varepsilon>0$,

$$\liminf_{T\to\infty}\frac{1}{T}\mathrm{meas}\left\{\tau\in[0,T]:\sup_{s\in K}|\zeta(s+i\tau)-f(s)|<\varepsilon\right\}>0.$$

In 2012, during the Conference in Astrakhan, Professor Barak Weiss posed a question does the upper limit can be replaced by the ordinary limit. The answer is the following: yes, except for at most countable set of values of $\varepsilon>0$.

The second remark is concerned to the number of zeros of certain compositions of universal zeta-functions.

The third remark is devoted a discrete universality of the Hurwitz zeta-function $\zeta(s,\alpha)$. Let $H(K)$, $K\in\mathcal{K}$, be the class of continuous functions on K which are analytic in the interior of K. Suppose that $0<\alpha\leq 1$ and $h>0$ are such that the set

$$\left\{(\log(m+\alpha):m\in\mathbb{N}\cup\{0\}),\tfrac{2\pi}{h}\right\}$$

is linearly independent over $\mathbb{Q}$. Let $K\in\mathcal{K}$ and $f(s)\in H(K)$. Then, for every $\varepsilon>0$,

$$\liminf_{T\to\infty}\frac{1}{N+1}\#\left\{0\leq m\leq N:\sup_{s\in K}|\zeta(s+imh,\alpha)-f(s)|<\varepsilon\right\}>0.$$

Vsevolod Lev (Haifa, Israel)

$a + b = 2c$

For a finite real set C of given size, the number of three-term arithmetic progressions in C is maximized when C itself is an arithmetic progression. Suppose now that C is split into two subsets, say A and B, and only those progressions with the smallest element in A, and the largest element in B, are counted; how many such "scattered progressions" can there be? Yet more generally, suppose we want to find two (possibly, intersecting) sets of real numbers, A and B, with $|A| + |B|$ prescribed, to maximize the number of three-term progressions with the smallest element in A, the largest element in B, and the middle element in $A \cup B$; how should A and B be chosen? We answer this and several related questions.

Joint work with Rom Pinchasi.

Renata Macaitienė (Šiauliai, Lithuania)

On the mixed joint universality

After a remarkable Voronin's work [1] on the universality of the Riemann zeta-function $\zeta(s)$ $(s = \sigma + it)$, it is known that the majority of other zeta and L-functions also are universal in the sense that their shifts approximate uniformly on compact subsets of certain regions wide classes of analytic functions. In the report, some questions concerning the joint universality will be analysed, more precisely, we will discuss the simultaneous approximation of a collection of any finite sets of analytic functions by shifts of different in some sense functions.

Let $\chi(m)$ be a Dirichlet character mod q, and let $L(s, \chi)$ be the corresponding L-function. Further, let $\lambda \in \mathbb{R}$ and $0 < \alpha \leq 1$, then the Lerch zeta-function $L(\lambda, \alpha, s)$, for $\sigma > 1$, is defined by

$$L(\lambda, \alpha, s) = \sum_{m=1}^{\infty} \frac{\mathrm{e}^{2\pi i \lambda m}}{(m+\alpha)^s},$$

and by analytic continuation elsewhere.

It is well known that the functions $L(s, \chi)$ and $L(\lambda, \alpha, s)$ (for some parameters λ and α) are universal. Moreover, these functions are also jointly universal. We will focus on the following result.

Theorem [2]. *Suppose that $\chi_1, ..., \chi_l$ are pairwise non-equivalent Dirichlet characters, and that the set $L(\mathcal{P}, \alpha_1, ..., \alpha_r) = \{(\log p : p \in \mathcal{P}), (\log(m + \alpha_j) : m \in \mathbb{N}_0,\ j = 1, ..., r)\}$ is linearly independent over $\mathbb{Q}$. For $j = 1, ..., l$, let $K_j \subset D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ be a compact set with connected complement and $f_j(s)$ be a continuous non-vanishing function on K_j which is analytic in the interior of K_j. Moreover, for $j = 1, ..., r$, let $\widehat{K}_j \subset D$ be a compact set with connected complement and $\widehat{f}_j(s)$ be a continuous function on $\widehat{K}_j$ and analytic in the interior of $\widehat{K}_j$. Then, for every $\epsilon > 0$ and $\lambda_j \in (0, 1]$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas}\left\{ \tau \in [0, T] : \sup_{1\leq j\leq l} \sup_{s\in K_j} |L(s + i\tau, \chi_j) - f_j(s)| < \varepsilon, \right.$$

$$\sup_{1\le j\le r}\sup_{s\in\widehat{K}_j}|L(\lambda_j,\alpha_j,s+i\tau)-\widehat{f}_j(s)|<\varepsilon\Bigg\}>0.$$

We observe that Dirichlet L-functions have Euler product over primes while Lerch zeta-functions do not have Euler product, except for the cases $\zeta(1,1,s)=\zeta(s)$ and $\zeta(1,1/2,s)=(2^s-1)\zeta(s)$. Thus, this theorem joins the universality and the strong universality.

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Kohji Matsumoto (Nagoya, Japan)

A numerical study on the behaviour of the Euler double zeta-function

This is a joint work with Mayumi Shōji (Japan Women's University).

Let

$$\zeta_2(s_1, s_2) = \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \frac{1}{n_1^{s_1}(n_1+n_2)^{s_2}}$$

be the Euler double zeta-function, where s_1, s_2 are complex variables. It is known that this function can be continued meromorphically to the whole space $\mathbb{C}^2$.

In this talk I will report some results on numerical study on the behaviour of $\zeta_2(s_1, s_2)$, especially its distribution of zeros.

First consider the case $s_1 = s_2 (= s)$. Then our function is a function of one-variable, and we can see that the distribution of zeros of $\zeta_2(s, s)$ has some similarity to that of zeros of Hurwitz zeta-functions. For example, we find a phenomenon which shares a common feature with a result of R. Garunkštis and J. Steuding (Analysis **22** (2002), 1-12) on the zeros of Hurwitz zeta-functions.

I will also report our attempt to trace the behaviour of zeros of $\zeta_2(s_1, s_2)$ as a function of two-variables.

Hidehiko Mishou (Tokyo, Japan)

A new method to prove joint universality for zeta functions

In 1980s, S. M. Gonek, B. Bagchi, and S. M. Vornoin independently proved the joint universality theorem for Dirichlet L-functions. In their proofs, the periodicity of Dirichlet characters and the orthogonality of characters play essential roles.

Recently I succeed to establish a new method to prove joint universality for zeta functions which does not depend on the periodicity of the coefficients of the zeta functions. Namely, I proved the joint universality theorems for the following sets of zeta functions:

1. A Pair of some types of zeta functions associated with holomorphic Hecke eigen cusp forms.
2. A set of Lerch zeta functions $\{L(\lambda_i, \alpha, s) \mid 1 \leq i \leq r\}$, where α is a real transcendental number and λ_j's are real algebraic irrational numbers such that $1, \lambda_1, \ldots, \lambda_r$ are linearly independent over $\mathbb{Q}$.
3. The Riemann zeta function $\zeta(s)$ and two twisted automorphic L-functions $L(s, f_j, \chi_j)$ $(j = 1, 2)$, where f_j are Hecke eigen cusp forms and χ_j (mod q_j) are non-equivalent Dirichlet characters.

Nikolay Moshchevitin (Moscow, Russia)

On a question by N. Chevallier

We solve a recent problem by N. Chevallier [C], concerning simultaneous Diophantine approximation vectors which form an unimodilar matrix.

Theorem. *Given function $\varphi(t)$ decreasing to zero as $t \to \infty$, there exists a vector $(1, \xi_1, \xi_2) \in \mathbb{R}^3$ with linearly independent over $\mathbb{Z}$ components and such that for any unimodular matrix*

$$\begin{pmatrix} q_1 & q_1 & q_3 \\ a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$$

one has

$$\max_{i=1,2,3} \left(\frac{\max_{j=1,2} |q_i \xi_j - a_{j,i}|}{\varphi(q_i)} \right) \geq 1.$$

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Radhakrishnan Nair (Liverpool, UK)

Ergodic Methods for Continued Fractions

The study of the statistical properties of continued fractions was initiated by C. F. Gauss . Building on subsequent of R. O. Kuzmin, A. Khinchin and W. Doeblin, C Ryll-Nardzewski showed that this theory could be based on Birkhoff's pointwise ergodic theorem. This has evolved into a substantial field called the metric theory of algorithms, covered by 11K in Mathematics Reviews. In this talk I will describe this subject and a portion of its modern development. Topics covered might include, The Euclidean Algorithm, The Gauss Map, The invariant measure, ergodicity and mixing, Birkhoff's pointwise ergodic theory, subsequence ergodic theory, the natural extention map, Hurwitz constants, entropy, the Bernoulli shift, Markov maps of the unit interval, invariant measures for Markov maps, the set of badly approximable points, relatives of the continued fraction maps – the nearest integer continued fraction expansion. Non-archemedian continued fractions, the continued fraction expansion in positive characteristic and the Schneider p-adic continued fraction expansion.

Takashi Nakamura (Tokyo, Japan)

Zeta distributions and quasi-infinite divisibility

Quasi-infinitely divisible characteristic functions are the quotients of two infinitely divisible characteristic functions. We give a quasi-infinitely divisible characteristic function $g(t)$ such that $(g(t))^u$ is not a characteristic function for any $u \in \mathbb{R}$ except for non-negative integers. We also give a not infinitely divisible but quasi-infinitely divisible characteristic function on $\mathbb{R}^2$. Zeta distributions play an important role in the proofs.

Ryotaro Okazaki (Kyoto, Japan)

Totally imaginary quartic Thue equation with three solutions

Let F be a totally imaginary quartic form with integer coefficients. Denote by N_F the number of solutions to the Thue equation $F(x,y)=1$, where we count modulo (x,y) and $(-x,-y)$ as one solution.

Nagell determined classes of F with $N_F \geq 3$ under the condition that the field generated by a root of $F(X,1)=1$ contains a quadratic field. In this talk, we drop that condition by using an algebraic invariants of the form F.

Tomokazu Onozuka (Nagoya, Japan)

The multiple Dirichlet product and the multiple Dirichlet series

First, we define the multiple Dirichlet product and study the properties of it. From those properties, we obtain a zero-free region of a multiple Dirichlet series and a multiple Dirichlet series expression of the reciprocal of a multiple Dirichlet series.

Liudmila Ostroumova, Egor Samosvat (Moscow, Russia)

Recency-based preferential attachment models

Preferential attachment models were shown to be very effective in predicting such important properties of real-world networks as the power-law degree distribution, small diameter, etc. Many different models are based on the idea of preferential attachment: LCD, Buckley-Osthus, Holme-Kim, fitness, random Apollonian network, and many others.

Although preferential attachment models reflect some important properties of real-world networks, they do not allow to model the so-called *recency property*. Recency property reflects the fact that in many real networks nodes tend to connect to other nodes of similar age. This fact motivated us to introduce a new class of models – recency-based models. This class is a generalization of fitness models, which were suggested by Bianconi and Barabási. Bianconi and Barabási extended preferential attachment models with pages' inherent quality or *fitness* of nodes. When a new node is added to the graph, it is joined to some already existing nodes that are chosen with probabilities proportional to the product of their fitness and incoming degree.

We generalize fitness models by adding a recency factor to the attractiveness function. This means that pages are gaining incoming links according to their *attractiveness*, which is determined by the incoming degree of the page, its *inherent popularity* (some page-specific constant) and age (new pages are gaining new links more rapidly).

We analyze different properties of recency-based models. For example, we show that some distributions of inherent popularity lead to the power-law degree distribution.

Vladimir Parusnikov (Moscow, Russia)

The Lagrange theorem for continued fractions of inhomogeneous linear forms[1]

Two real numbers $0 \leq \alpha < 1, 0 \leq \beta < 1$ define at points $(y, z) \in \mathbb{R}^2$ the inhomogeneous linear form $L_{\alpha,\beta}(y, z) = -\beta + \alpha y + z$. We propose the algorithm of an expansion of this linear form into the inhomogeneous continued fraction, that will be designated as

$$L_{\alpha,\beta} \sim [\ 0\ ;\ b_1, b_2, \ldots] \bmod [\ 0\ ;\ a_1, a_2, \ldots].$$

Inhomogeneous continued fraction generalizes the classic regular continued fraction: for $\beta = 0$ all the numbers $b_n = 0$, and we get the continued fraction expansion of the number α: $L_{\alpha,0} \sim [\ 0\ ] \bmod [\ 0\ ;\ a_1, a_2, \ldots]$.

For inhomogeneous continued fractions the analog of the Lagrange Theorem is valid:

Theorem 2. *Inhomogeneous continued fraction is periodic if and only if α is quadratic irrationality with positive discriminant and β is linear expression $\beta = r\alpha + s$ with rational $r, s \in \mathbb{Q}$.*

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Rom Pinchasi *(Haifa, Israel)*

On the union of arithmetic progressions

We show that for every $\epsilon > 0$ there is an absolute constant $c(\epsilon) > 0$ such that the following is true: The union of any n arithmetic progressions, each of length n, with pairwise distinct differences must consist of at least $c(\epsilon)n^{2-\epsilon}$ elements. We show also that this type of bound is essentially best possible, as we can find n arithmetic progressions, each of length n, with pairwise distinct differences such that the cardinality of their union is $o(n^2)$.

We develop some number theoretical tools that are of independent interest. In particular we give almost tight bounds on the following question: Given n distinct integers $a_1, ..., a_n$ at most how many pairs satisfy $a_j/a_i \in [n]$? More tight bounds on natural related problems will be presented.

This is a joint work with Shoni Gilboa.

A.M. Raigorodskii[1] *(Moscow, Russia)*

Forbidding distances on $(0,1)$- and $(-1,0,1)$-spaces: new results and their applications in combinatorial geometry[2]

Consider a set $V \subseteq \{0,1\}^n$. Let $W \subseteq V$ be such that for any two elements $\mathbf{x}, \mathbf{y} \in W$ the distance between them does not belong to a prescribed set $\mathcal{A}$ of real numbers. The classical question is in finding the maximum cardinality of W depending on $\mathcal{A}$. Many deep results in this area have been obtained during the last 30 years, and many problems still remain open.

Another more general question concerns the case when $V \subseteq \{-1,0,1\}^n$. Here the number of important unsolved problems is even much bigger than in the (0,1)-case. Moreover, if in that case there is no difference between the Euclidean distance, the l_1-distance, and the Hamming distance, here already the differences are substantial.

One of the applications of the above-mentioned results is provided by combinatorial geometry. The point is that we may consider V as the set of vertices of a distance graph in $\mathbb{R}^n$ whose edges are among pairs $\{\mathbf{x}, \mathbf{y}\}$ where $\mathbf{x}$ and $\mathbf{y}$ are at a given distance apart. This interpretation helps in obtaining multiple strong corollaries in, say, Nelson–Hadwiger problem concerning the so-called chromatic numbers of spaces or in Borsuk's problem on set decompositions into parts of smaller diameters.

In our talk, we shall give a survey of some classical and new results in the area. We shall also discuss some very recent achievements and conjectures. We shall finally explain how do they influence combinatorial geometry.

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[1]Moscow State University, Mechanics and Mathematics Faculty, Department of Mathematical Statistics and Random Processes; Moscow Institute of Physics and Technology, Faculty of Innovations and High Technology, Department of Discrete Mathematics; Yandex research laboratories.

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Irina Rezvyakova *(Moscow, Russia)*

Zeros of the Epstein zeta function on the critical line

In the late 90s Atle Selberg invented a new method allowed one to prove under certain natural conditions that a general linear combination of L-functions contains a positive proportion of its nontrivial zeros on the critical line. In our talk we shall provide the details of this result for the Epstein zeta function of the positive definite binary quadratic form.

Alexandr Sapozhenko[1] *(Moscow, Russia)*

On the number of sets free from solutions of linear equations in abelian groups

A survey of results concerning the sets without solutions of linear equations is represented. In particular, we consider sum-free sets, sets without zeros, (k, l) sum-free sets. The methods of estimation are discussed.

[1]Research supported by RFBR grant No 13-01-00586.

Alexandr Sapozhenko[1] *(Moscow, Russia)*

О максимальной длине сокращенной ДНФ

Элементарная конъюнкция K называется импликантой булевой функции f, если справедливо соотношение $K \bigwedge f = K$. Импликанта K функции f называется простой, если всякая конъюнкция $L \neq K$, такая, что $K \bigwedge L = K$, не является импликантой функции f. Дизъюнкция всех простых импликант функции f называется ее сокращенной дизъюнктивной нормальной формой (сокращенной ДНФ). Число простых импликант функции f называется длиной сокращенной ДНФ и обозначается в дальнейшем через $s(f)$. Положим $s(n) = \max s(f)$, где макимум берется по всем булевым функциям от n аргументов. Задача о значении величины $s(n)$ была поставлена С.В.Яблонским в начале 60-х годов прошлого века. А.П.Викулин [1] нашел максимум величины $s(f)$ для симметрических функций. Это дает нижнюю оценку вида $3^n/n$ для $s(n)$. Верхняя оценка оценка вида $s(n) \leq 3^n/\sqrt{n}$, также полученная в [1], получается также из тех соображений, что множество граней любой сокращенной ДНФ представляет собой антицепь в частично упорядоченном (по включению) множестве G^n всех граней n-мерного куба. Оценка мощности антицепи приводит к той же верхней оценке, что получена Викулиным (см., например, [2]). Верхняяи нижняя оценки Викулина отличаются по порядку в $\sqrt{n}$ раз. Целью данного сообщения является следующая

Theorem 3. *Существует положительная константа c, такая, что*

$$s(n) \geq c3^n/\sqrt{n}.$$

Тем самым получен порядок величины $s(n)$. Доказательство опирается на результат И. П. Чухрова, доказавшего существование антицепи, тень которой по порядку равна 2^n (см.[3]).

[1]Работа выполнена при финансовой поддержке РФФИ, проект №13–01–00958-а.

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Paulius Šarka *(Vilnius, Lithuania)*

Least common multiple of random sets

In this talk we will present our results concerning a random variable

$$\varphi(n, \delta) = \log \operatorname{lcm}[n]_\delta,$$

where by $[n]_\delta$ *we denote a random subset of an interval* $[n]$*, with elements selected independently with probability* δ*. We estimate its asymptotic distribution when* $n \to \infty$*, and discuss connection with recent results on the asymptotics of generalized Chebyshev's function.*

This is a joint work with Javier Cilleruelo, Juanjo Rué and Ana Zumalacárregui.

Ilya Shkredov *(Moscow, Russia)*

The eigenvalues method in Combinatorial Number Theory

In the talk a family of operators (finite matrices) with interesting properties will be discussed. These operators appeared during attempts to give a simple proof of Chang's theorem from Combinatorial Number Theory. At the moment our operators have found several applications in the area connected with Chang's result as well as another problems of Number Theory such as : bounds for the additive energy of some families of sets, new structural results for sets with small higher energy, estimates of Heilbronn's exponential sums and others.

Darius Šiaučiūnas *(Šiauliai, Lithuania)*

Universality of composite functions of periodic zeta-functions

In the report, we consider the universality of the function $F(\zeta(s;\mathfrak{a}),\zeta(s,\alpha;\mathfrak{b}))$, *where* $\zeta(s;\mathfrak{a})$ *and* $\zeta(s,\alpha;\mathfrak{b})$, $s=\sigma+it$, *are the periodic and periodic Hurwitz zeta-functions, respectively, defined, for* $\sigma>1$, *by the series*

$$\zeta(s;\mathfrak{a})=\sum_{m=1}^{\infty}\frac{a_m}{m^s} \qquad \text{and} \qquad \zeta(s,\alpha;\mathfrak{b})=\sum_{m=0}^{\infty}\frac{b_m}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. Here $\mathfrak{a}=\{a_m\}$ *and* $\mathfrak{b}=\{b_m\}$ *are periodic sequences of complex numbers, and* $0<\alpha\leq 1$ *is a fixed parameter.*

Let $H(D)$, $D=\left\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\right\}$, *be the space of analytic functions on* D. *We say that the operator* $F:\ H^2(D)\to H(D)$ *belongs to the class* $Lip(\beta_1,\beta_2)$, $\beta_1,\beta_2>0$, *if the following hypotheses are satisfied:*

1° *For any compact set* $K\subset D$ *with connected complement and each polynomial* $p=p(s)$, *there exists an element* $(g_1,g_2)\in F^{-1}\{p\}\subset H^2(D)$ *such that* $g_1(s)\neq 0$ *on* K;

2° *For any compact set* $K\subset D$ *with connected complement, there exist a constant* $c>0$ *and compact sets* $K_1,K_2\subset D$ *with connected complements such that*

$$\sup_{s\in K}|F(g_{11}(s),g_{12}(s))-F(g_{21}(s),g_{22}(s)))|\leq c\sup_{1\leq j\leq 2}\sup_{s\in K_j}|g_{1j}(s)-g_{2j}(s)|^{\beta_j}$$

for all $(g_{j1},g_{j2})\in H^2(D)$, $j=1,2$. *Also, by* $\mathrm{meas}\{A\}$ *denote the Lebesgue measure of a measurable set* $A\subset\mathbb{R}$. *Then we have the following result.*

Theorem 1. *Suppose that the* α *is a transcendental number, and that* $F\in Lip(\beta_1,\beta_2)$. *Let* $K\subset D$ *be a compact set with connected complement, and* $f(s)$ *be a continuous function on* K *which is analytic in the interior of* K. *Then, for every* $\varepsilon>0$,

$$\liminf_{T\to\infty}\frac{1}{T}\mathrm{meas}\left\{\tau\in[0,T]:\sup_{s\in K}\left|F\left(\zeta(s+i\tau;\mathfrak{a}),\zeta(s+i\tau,\alpha;\mathfrak{b})\right)-f(s)\right|<\varepsilon\right\}>0.$$

The details and other results can be found in [1].

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Teerapat Srichan *(Würzburg, Germany)*

Sampling the Lindelöf Hypothesis for Hurwitz zeta-functions by a Cauchy random walk

We study the behaviour of the Hurwitz zeta-function $\zeta(\frac{1}{2}+it,a)$, when $0 < a \leq 1$ and t is sampled by the Cauchy random walk. Let $X_1, X_2, ...$ denote an infinite sequence of independent Cauchy-distributed random variables. Consider the sequence of partial sums $S_n = X_1 + ... + X_n$, $n = 1, 2, ...$ We investegete the almost-sure asymptotic behaviour of the system $\zeta(\frac{1}{2}+iS_n,a)$, $n = 1, 2, ...$ We develop a complete second-order theory for this system and show, by using a approximation formula of $\zeta(\bullet,a)$, that it behaves almost like a system of non-correlated variables. Exploting this fact in relation with the know criteria for almost-sure asymptotic behaviour: for any real $b > 2$ it is true that $\sum_{k=1}^{n}\zeta(\frac{1}{2}+iS_k,a) = n + O(n^{\frac{1}{2}}\log(1+n)^b)$ almost surely.

Jörn Steuding *(Würzburg, Germany)*

One Hundred Years Uniform Distribution Modulo One and Recent Applications to Riemann's Zeta-Function

The theory of uniform distribution modulo one has been founded by Weyl and others around one hundred years ago. Following Rademacher and Hlawka, we show that the ordinates of the nontrivial zeros of the zeta-function $\zeta(s)$ *are uniformly distributed modulo one. We conclude with recent investigations concerning the distribution of the roots of the equation* $\zeta(s) = a$*, where* a *is any complex number.*

Masatoshi Suzuki *(Tokyo, Japan)*

An inverse problem for a class of canonical systems and its applications

A canonical system is a first-order system of ordinary differential equations parametrized by all complex numbers. Like Sturm-Liouville equations, string equations and Dirac type systems, a number of second-order differential equations and systems of first-order differential equations are reduced to canonical systems. It is known that any solution of a canonical system generates an entire function of the Hermite-Biehler class. An inverse problem is to recover a canonical system from a given entire function of the Hermite-Biehler class satisfying appropriate conditoins. This type inverse problem was solved by de Branges in 1960s. However his results are often not enough to investigate a Hamiltonian of recovered canonical system. In this talk, we present an explicit way to recover a Hamiltonian from a given exponential polynomial belonging to the Hermite-Biehler class. After that, we apply it to study distributions of roots of self-reciprocal polynomials.

László Tóth *(Wien, Austria and Pécs, Hungary)*

On the number of subgroups of finite Abelian groups of rank two and three

Let $\mathbb{Z}_m$ denote the additive group of residue classes modulo m. For positive integers m and n consider the direct product $\mathbb{Z}_m \times \mathbb{Z}_n$, which has rank two in the case $\gcd(m,n) > 1$. Let $c(m,n)$ and $s(m,n)$ be the number of cyclic subgroups and the number of all subgroups of $\mathbb{Z}_m \times \mathbb{Z}_n$, respectively.

For every $m, n \geq 1$,

$$c(m,n) = \sum_{d \mid m, e \mid n} \phi(\gcd(d,e)) = \sum_{d \mid \gcd(m,n)} \phi(d) \tau(mn/d^2),$$

and

$$s(m,n) = \sum_{d \mid m, e \mid n} \gcd(d,e) = \sum_{d \mid \gcd(m,n)} d\, \tau(mn/d^2),$$

where ϕ is Euler's function and τ stands for the divisor function.

Furthermore,

$$\sum_{m,n \leq x} s(m,n) = x^2 \left(\frac{2}{\pi^2} \log^3 x + c_1 \log^2 x + c_2 \log x + c_3 \right) + \mathcal{O}\left(x^{1117/701} \right),$$

where c_1, c_2, c_3 are constants and $1117/701 \approx 1.5934$.

A similar formula holds for $\sum_{m,n \leq x} c(m,n)$.

In the talk, based on the papers [1, 2, 3, 4] I will discuss these and other results regarding $c(m,n)$ and $s(m,n)$. The number of (cyclic) subgroups of $\mathbb{Z}_m \times \mathbb{Z}_n \times \mathbb{Z}_r$ will also be considered.

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Andrey Voronenko *(Moscow, Russia)*

On one property of almost all Boolean functions

A new problem is considered. Alice has an opportunity to convince Bob with her function has some property. But it is false suggestion. So Alice has to make Bob be sure she has some definite function. Bob can calculate it precisionly. Alice can give real values of function. Alice can either construct a function or use a random one. Both variants are considered. In the paper we study the linearity property.

Consider the following situation. Alice somehow managed to convince Bob she posesses a linear ($\alpha \oplus \alpha_1 x_1 \oplus \ldots \oplus \alpha_n x_n$) Boolean function. In practice, Alice can choose any partical function she wants. The purpose of Alice is for any linear function to make Bob be sure she has that function, giving him values of her real function. If her partial function enables to do that it is said to be universal. The linear upper bound for the range of definition of universal functions was obtained in [1]. Paper [2] is devoted to analysis of the false monotonicity property.

Let universal function $f(x_1, \ldots, x_k)$ be defined on m sets. Then we say $c = \frac{m-1}{k}$ is the working constant for f.

Theorem 1. *For any universal function $f(x_1, \ldots, x_k)$ and for any l there is a universal function h, depending on lk arguments with the same working constant as f.*

Theorem 2. *Suppose there is a universal function $f(x_1, \ldots, x_k)$, defined on m sets. Then for any ε the probability of arbitrary function $g(x_1, \ldots, x_n)$ has a universal subfunction, defined not more than on $\frac{(m-1)n}{k^2}(1+\varepsilon)$ sets tends to 1 when $n \to \infty$.*

It was proved [1] that all universal functions depend on more than three arguments. Examples of universal functions, depending of 4 and 5 arguments were also shown in [1]. A universal function, depending on six arguments defined on twenty sets (working constant $3\frac{1}{6}$) was constructed in [3].

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Vytas Zacharovas *(Vilnius, Lithuania)*

Limit distribution of the coefficients of polynomials with only unit roots

We investigate the limiting distribution of a sequence of random variables whose generating functions are polynomials whose all roots are located on the unit circle.

The main result of our paper [1] is the necessary and sufficient conditions under which the sequence of such variables converges to normal distribution. Moreover we also derive a representation theorem for all possible limit laws and apply our results to many concrete examples in the literature, ranging from combinatorial structures to numerical analysis, and from probability to analysis of algorithms.

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Maksim E. Zhukovskii *(Moscow, Russia)*

Critical points in zero-one laws for $G(n,p)$

We study asymptotical behaviour of the probabilities of first-order properties for Erdös–Rényi random graphs $G(n,p)$. It was proved by Y.V. Glebskii, D.I. Kogan, M.I. Liagonkii and V.A. Talanov [1] in 1969 (and independently in 1976 by R. Fagin [2]) that for any first order property L either "almost all" graphs satisfy this property as N tends to infinity or "almost all" graphs don't satisfy the property. In other words, if p doesn't depend on N, then for any first-order property L either the random graph satisfies the property L almost surely or it doesn't satisfy (in such cases the random graph is said to obey zero-one law*). We consider the probabilities $p = p(n)$, where $p(n) = n^{-\alpha}$, $n \in \mathbb{N}$, for $\alpha \in (0,1)$. The zero-one law for such probabilities was proved by S. Shelah and J.H. Spencer [3]. When $\alpha \in (0,1)$ is rational the zero-one law in ordinary sense for these graphs doesn't hold.*

Let k be a positive integer. Denote by $\mathcal{L}_k$ the class of the first-order properties of graphs defined by formulae with quantifier depth bounded by the number k (the sentences are of a finite length). Let us say that the random graph obeys zero-one k-law*, if for any first-order property $L \in \mathcal{L}_k$ either the random graph satisfies the property L almost surely or it doesn't satisfy. We study the set $Z(k) \subset [0,1]$ of such α that $G(n, n^{-\alpha})$ does not obey zero-one k-law.*

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Victoria Zhuravleva *(Moscow, Russia)*

On the distribution of powers of some Pisot numbers[1]

Put $L(\theta) = \sup_{\xi\in\mathbb{R}} \liminf_{n\to\infty} ||\xi\theta^n||$ *(here* θ *is a real number,* $||\cdot||$ *is the distance to the nearest integer). In this talk we present some new results for algebraic numbers* θ *of degree* ≤ 4.

1) If θ *is the largest root of* $x^2 - ax + b = 0$, *where* $a, b \in \mathbb{N}$, $a > b + 1$, $a + b$ *is even, then* $L(\theta) = \frac{a+b}{2(a+b)+2}$.

2) If θ *is the largest root of* $x^3 - x - 1 = 0$, *then* $L(\theta) = \frac{1}{5}$.

3) If θ *is the largest root of* $x^4 - x^3 - 1 = 0$, *then* $L(\theta) = \frac{3}{17}$.

We should notice that all the numbers θ *from 1), 2), 3) are Pisot numbers, while those from 2) and 3) are the smallest Pisot numbers.*

[1]Supported by RFBR grant No. 12-01-00681-a