Vilnius Conference in Combinatorics and Number Theory

Vilnius, Lithuania, July 16 - July 22, 2017

Program and Abstract Book

List of Participants

Renat Akhunzhanov akhunzha@mail.ru
Sebastian Alterman szalterman@gmail.com
Dmitriy Bilyk bilyk.dmitriy@gmail.com
Thomas Bloom warehouse23@gmail.com
Florin Boca fboca@illinois.edu
Rafał Bystrzycki rafbys@amu.edu.pl
Pablo Candela pablo.candela@uam.es
Henrietta Dickinson detta.dickinson@maths.nuim.ie
Valdas Dičiūnas valdas.diciunas@mif.vu.lt
Edward Dobrowolski Edward.Dobrowolski@unbc.ca
Lenny Fukshansky lenny@cmc.edu
Dmitry Gayfulin gamak.57.msk@gmail.com
Jonas Jankauskas jonas.jankauskas@gmail.com
Gyula Katona katona.gyula.oh@renyi.mta.hu
Harald Andrés Helfgott harald.helfgott@gmail.com
Sandor Kiss kisspest@cs.elte.hu
Maxim Korolev hardy_ramanujan@mail.ru
Takeshi Kurosawa tkuro@rs.tus.ac.jp
Antanas Laurinčikas antanas.laurincikas@mif.vu.lt
Eugenijus Manstavičius eugenijus.manstavicius@mif.vu.lt
Antoine Marnat marnat@math.tugraz.at
Mate Matolcsi matemato@gmail.com
Stephen McGuire stephenmcguire07@hotmail.com
Dezső Miklós dezso.miklos@gmail.com
Nikolay Moshchevitin moshchevitin@gmail.com
Brendan Murphy Brendan.Murphy@bristol.ac.uk
Radhakrishnan Nair nair@liverpool.ac.uk
Peter Pal Pach p.p.pach@gmail.com
Fedor Petrov fedyapetrov@gmail.com
Roman Prosanov rprosanov@mail.ru
Joël Rivat joel.rivat@univ-amu.fr
Oliver Roche-Newton o.rochenewton@gmail.com
Anne de Roton anne.deroton@gmail.com
Misha Rudnev M.Rudnev@bristol.ac.uk
Johannes Schleischitz johannes.schleischitz@boku.ac.at
Tomasz Schoen schoen@amu.edu.pl
Alisa Sedunova alisa.sedunova@phystech.edu
Aliaksei Semchankau leshaxim@mail.ru
Alexandr Semenov alexsemenov1992@mail.ru
Dmitrii Shabanov gold-amber@yandex.ru
Talia Shaikheeva talia.shaikheeva@yandex.ru
Iekata Shiokawa shiokawa@beige.ocn.ne.jp
Ilya Shkredov ilya.shkredov@gmail.com
Darius Šiaučiūnas darius.siauciunas@su.lt
Raivydas Šimenas raivydas.simenas@mif.vu.lt

Jozsef Solymosi solymosi@math.ubc.ca
Gediminas Stepanauskas gediminas.stepanauskas@mif.vu.lt
Sophie Stevens Sophie_ccs.2011@my.bristol.ac.uk
Leonhard Summerer Leonhard.Summerer@univie.ac.at
Katarzyna Taczała katarzyna.taczala@amu.edu.pl
Rokas Tamošiūnas rokas.tamosiunas@mif.vu.lt
Jörg Thuswaldner jorgthus@gmail.com
Lola Thompson lola.thompson@oberlin.edu
Yohei Tachiya tachiya@hirosaki-u.ac.jp
Artem Uvakin artemuvakin@gmail.com
Anatoly Vershik vershik@pdmi.ras.ru
Carlo Viola viola@dm.unipi.it
Alexei Volostnov gyololo@rambler.ru
Dmitrii Zhelezov dzhelezov@gmail.com
Victoria Zhuravleva victoria.zhuravleva@me.com

SCHEDULE

Conference takes place at Department of Mathematics and Informatics of the University of Vilnius, Naugarduko str. 24, Vilnius, rooms 102 a, 103 a

Monday 17.07

09:30 - 10:10	**REGISTRATION**		
10:10 - 10.50	Gyula Katona	An improvement on the intersecting shadow theorem	
Coffee break			
11:20 - 12:00	Dmitry Shabanov	The Erdős–Hajnal problem on colorings of hypergraphs, its on-line generalizations and related questions	
Lunch break			
	SECTION A		SECTION B
14:00 - 14:30	Pablo Candela On sets with small sumset in the circle		Lola Thompson Divisor-sum fibers
14:40 - 15:10	Roman Prosanov Chromatic numbers of spaces and fractional covering technique		Alisa Sedunova A logarithmic improvement in the Bombieri-Vinogradov theorem
15:20 - 15:50	Fedor Petrov Group algebra and the structure of product set		Yohei Tachiya On algebraic relations between the values of the theta function
16:00 - 16:20	Rafał Bystrzycki Sums of dilates		Darius Šiašiūnas On weighted discrete universality of periodic zeta-functions
Coffee break			
16:50 - 17:10	Victoria Zhuravleva	Periodic sequences modulo 1 and Pisot numbers	
17:20 - 17:50	Thomas Bloom	Structure of spectra	
18:00 - 18:30	Lenny Fukshansky	On two constructions of extremal lattices	

Tuesday 18.07

09:30 - 10:10	Harald Andrés Helfgott	Voronoi, Sierpinski, Eratosthenes	
10:20 - 10.50	Maxim Korolev	On the estimates of Kloosterman sums	
Coffee break			
11:20 - 12:00	Joël Rivat	On the digits of primes and squares	
Lunch break			
	SECTION A		SECTION B
14:00 - 14:30	Sandor Kiss On the structure of sets which has coincide representation functions		Mate Matolcsi Colouring the plane with sets avoiding the unit distance
14:40 - 15:10	Radhakrishnan Nair Weyl's theorem on uniform distribution and ergodic theorems		Dezső Miklós The vertex and edge sign balance of (hyper)graphs
15:20 - 15:50	Dmitriy Bilyk Uniform distribution problems on the sphere		Alexander Semenov On the j-chromatic number of random hypergraphs
16:00 - 16:20	Valdas Dičiūnas On the number of odd primitive abundant numbers with five and six distinct prime factors		Talia Shaikheeva List chromatic numbers of the complete multi-partite hypergraphs
Coffee break			
16:50 - 17:30	Jonas Jankauskas	On Littlewood and Newman multiples of Borwein polynomials	
17:40 - 18:00	Gediminas Stepanauskas and Jonas Šiaulys	Limit distributions for some sets of additive functions	

Wednesday 19.07

09:30 - 10:10	Jozsef Solymosi	Sum-product bounds for complex matrices
10:20 - 10:50	Misha Rudnev	Counting distinct cross-ratios
Coffee break		
11:20 - 11:50	Jörg Thuswaldner	Discrepancy bounds for $\boldsymbol{\beta}$-adic Halton sequences
12:00 - 12:30	Florin Boca	Moments and Distribution of Eigenvalues in Large Sieve Matrices
	FREE AFTERNOON	
19:00	**CONFERFENCE PARTY** restaurant La Traviata in hotel Artis (http://www.artis.centrumhotels.com/en/restaurants)	

Thursday 20.07

09:40 - 10:10	Edward Dobrowolski	Mahler measure of polynomials with k nonzero terms
10:20 - 10.50	Iekata Shiokawa	Irrationality exponents of numbers related with Cahen's constant
Coffee break		
11:20 - 12:00	Carlo Viola	The multidimensional saddle-point method in Diophantine approximation
Lunch break		

	SECTION A	SECTION B
14:00 - 14:30	Johannes Schleischitz Classical exponents of Diophantine approximation	Sophie Stevens An incidence theorem in finite fields and applications
14:40 - 15:10	Leonhard Summerer Simultaneous approximation and associated spectra	Brendan Murphy Rich lines in general position
15:20 - 15:50	Antoine Marnat Transference between homogeneous and inhomogeneous Diophantine approximation on manifolds	Oliver Roche-Newton On the size of the set $AA + A$ over the reals
16:00 - 16:20	Dmitry Gayfulin Attainable numbers and the Lagrange spectrum	Alexei Volostnov Sums of multiplicative characters with additive convolutions

Coffee break		
16:50 - 17:10	Raivydas Šimenas	Zero free regions of the Lerch zeta-function
17:20 - 17:40	Rokas Tamosiunas	Symmetry of zeros of Lerch zeta-function for equal parameters
17:50 - 18:10	Ilya Shkredov	On the asymmetric sum-product phenomenon

Friday 21.07

09:40 - 10.10	Tomasz Schoen	A new upper bounds for additive Ramsey numbers
10:20 - 10:50	Dmitrii Zhelezov	Convex sets can have thin additive bases
Coffee break		
11:20 - 11:50	Anne de Roton	Small sumsets in $\mathbb{R}$
12:00 - 12:30	Peter Pal Pach	On progression-free sets via the polynomial method
Lunch break		
14:30 - 14:50	Antanas Laurinčikas	Joint discrete universality of zeta-functions of certain cusp forms
15:00 - 15:20	Eugenijus Manstavičius	On the order statistics of component sizes of a random combinatorial structure
15:30 - 15:50	Katarzyna Taczała	The degree of regularity of the equation $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i + b$
16:00 - 16:20	Aliaksei Semchankau	Maximal subsets avoiding arithmetic progressions in sets
Coffee break		
16:50 - 17:30	Anatoly Vershik	Combinatorics of infinite 15-game and random Young tableau
17:40 - 18:20	Nikolay Moshchevitin	On some open problems in Diophantine Approximation and related fields

ABSTRACTS

Dzmitry Badziahin, **Approximational properties of certain Mahler numbers**

In the talk we will present several results about irrationality exponents together with some more sensitive measures of irrationality for a class of Mahler numbers defined by infinite products. Many of the results from the talk were achieved with help of recently discovered recurrent formulae for the continued fraction of the corresponding Mahler functions.

Dmitriy Bilyk, **Uniform distribution problems on the sphere**

The talk will be devoted to new results and connections between different concepts of uniform distribution of points on the sphere, such as discrepancy, energy minimization, cubature formulas and designs, uniform tessellations, almost isometric embeddings etc. In particular, we shall discuss numerous recent versions, extensions, and generalizations of the so-called Stolarsky invariance principle – an identity, which connects optimal discrepancy and energy minimizing points on the sphere. We shall also provide applications of this principle to various problems of discrete geometry. Parts of this work have been done jointly with M. Lacey, R. Matzke, and F. Dai.

Florin Boca, **Moments and Distribution of Eigenvalues in Large Sieve Matrices**

The classical large sieve inequality provides an upper bound estimate for the largest eigenvalue of the N by N matrix A^*A, where A is a Vandermonde type matrix defined by roots of unity of order at most Q. This talk will discuss some aspects concerning the behavior of the eigenvalues of these matrices when $N \sim cQ^2$, with $Q \to infty$ and $c > 0$ constant. In particular, we establish asymptotic formulas for their moments, proving as a corollary the existence of a limiting distribution as a function of c, and answering in the affirmative a problem of Ramare. This is joint work with Maksym Radziwill.

Rafał Bystrzycki, **Sums of dilates**

We investigate the size of the sets $\lambda_1 \cdot A + \cdots + \lambda_h \cdot A$, where λ_i are integers. Specifically, we look for upper bounds in terms of the doubling constant $K = \frac{|A+A|}{|A|}$. We also examine some situations in which those bounds can be significantly strengthened. Joint work with Tomasz Schoen.

Pablo Candela, **On sets with small sumset in the circle**

We shall discuss recent results concerning subsets of the circle group with doubling constant less than 3, including partial progress towards an analogue in this setting of Freiman's $3k - 4$ conjecture. We shall also discuss applications of these results to the problem of estimating how large can the measure of a subset of the circle be if the set avoids solutions to an equation of the form $x + y = mz$, for $m > 2$ an integer.

Valdas Dičiūnas, **On the number of odd primitive abundant numbers with five and six distinct prime factors**

Let $\mathrm{opa}(n)$ = number of odd primitive abundant numbers with n distinct prime factors, $\mathrm{opa} : \mathbb{N}^+ \to \mathbb{N}$. It has long been known that $\mathrm{opa}(1) = \mathrm{opa}(2) = 0$, $\mathrm{opa}(3) = 8$ and $\mathrm{opa}(4) = 576$ (Dickson 1913, Am. J. Math. 35(4), pp. 413–422). Unfortunately, even the bounds for the values $\mathrm{opa}(5)$ and $\mathrm{opa}(6)$ were not known (see sequence A188439 in OEIS, http://oeis.org/A188439). In our talk we present a method of generation of all odd primitive abundant numbers with a fixed number of distinct prime factors. In the case of five factors we prove that $\mathrm{opa}(5) = 3{,}913{,}172$. Generation of odd primitive abundant numbers with six distinct prime factors is still in progress. Up to now more than 50 trillions of such numbers were found, so $\mathrm{opa}(6) > 5 \cdot 10^{13}$.

Our method consists of reduction the problem of search in a sequence of infinite lattices of integers (ordered by division) to a problem of search in directed acyclic multigraphs. Then we apply the backtracking combined with the kNN method. We hope that our technique may be applied to generate other combinatorial objects as well.

Edward Dobrowolski, **Mahler measure of polynomials with k nonzero terms**

I would like to revisit my 2006 paper on Mahler measure of polynomials in terms of the number of its coefficients and the structure of polynomials that have large cyclotomic part, and present some simplifications in the proof that may lead to a sharpening of the lower bound of the measure.

Lenny Fukshansky, **On two constructions of extremal lattices**

A lattice in a Euclidean space is called extremal if it is a local maximum of the packing density function on the space of lattices in its dimension. A classical criterion due to Voronoi gives a beautiful geometric characterization of extremal lattices in terms of certain symmetric properties of their sets of minimal vectors. In this talk, we will describe two different constructions of lattices with interesting geometric properties. We will use Voronoi's criterion to exhibit families of extremal lattices coming from these constructions.

Dmitry Gayfulin, **Attainable numbers and the Lagrange spectrum**

Consider an arbitrary irrational number α having uniformly bounded partial quotients. The Lagrange constant $\mu(\alpha)$ is defined as follows

$$\mu(\alpha) = \left(\liminf_{p\in\mathbb{Z}, q\in\mathbb{N}} |q(q\alpha - p)| \right)^{-1}.$$

The set of all values taken by $\mu(\alpha)$ as α varies is called the *Lagrange spectrum* $\mathbb{L}$. Irrational α is called *attainable* if the inequality

$$\left| \alpha - \frac{p}{q} \right| \leqslant \frac{1}{\mu(\alpha) q^2}$$

holds for infinitely many relatively prime integers p and q. We call the Lagrange spectrum element λ *admissible* if there exists an irrational number α such that $\mu(\alpha) = \lambda$. In the paper [G1] it was found that there exist non-admissible elements in $\mathbb{L}$.

Theorem 1

The quadratic irrationality $\lambda_0 = [3; 3, 3, 2, 1, \overline{1, 2}] + [0; 2, 1, \overline{1, 2}]$ *belongs to* $\mathbb{L}$*, but is not an admissible number.*
My talk is devoted to the recently established [G2] sufficient and necessary criteria of admissibility of the Lagrange spectrum element.

Theorem 2

(i) The Lagrange spectrum element a is not admissible if and only if the following two conditions hold:
(i) The interval $(a, a + \varepsilon)$ *does not contain the elements of* $\mathbb{L}$ *for* ε *small enough.*
(ii) There does not exists a quadratic irrationality α *such that* $\mu(\alpha) = a$.

References

[G1] Gayfulin D. *Attainable numbers and the Lagrange spectrum*, Acta Arithmetica, to be published in 2017, DOI: 10.4064/aa8588-12-2016, preprint available at arXiv:https://arxiv.org/abs/1606.01600 (2016), 14 p.
[G2] Gayfulin D. *Admissible endpoints of gaps in the Lagrange spectrum*, preprint available at https://arxiv.org/abs/1704.08060

Harald Andrés Helfgott, **Voronoi, Sierpinski, Eratosthenes**

We show how to carry out a sieve of Erastosthenes up to N in space $O(N^{1/3})$ and essentially linear time. This improves over the usual versions, which take space about $O(\sqrt{N})$ and essentially linear time. The algorithm – which, like the one in (Galway, 2000), is ultimately related to diophantine approximation – can also be used to factorize integers n, and thus to give the values of arithmetical functions such as the Möbius function μ and the Liouville function λ for all integers up to N.

Jonas Jankauskas, Paulius Drungilas, and J. Šiurys, **On Littlewood and Newman multiples of Borwein polynomials**

A Newman polynomial has all the coefficients in $\{0, 1\}$ and constant term 1, whereas a Littlewood polynomial has all coefficients in $\{-1, 1\}$. We call $P(X) \in \mathbb{Z}[X]$ a *Borwein* polynomial if all its coefficients belong to $\{-1, 0, 1\}$ and $P(0) \neq 0$.

We exploit an algorithm developed previously by Frougny, Lau and Stankov in their research on *the spectra of numbers* and independently by Akiyama, Thuswaldner and Zaïmi in their study of *Height Reducing Property*. The algorithm decides whether a given monic integer polynomial with no roots on the unit circle $|z| = 1$ has a non-zero multiple in $\mathbb{Z}[X]$ with coefficients in a finite set $\mathcal{D} \subset \mathbb{Z}$.

Our results are as follows. For every Borwein polynomial of degree ≤ 9 we determine whether it divides any Littlewood or Newman polynomial. We show that every Borwein polynomial of degree ≤ 8 which divides some Newman polynomial divides some Littlewood polynomial as well. For every Newman polynomial of degree ≤ 11, we check whether it has a Littlewood multiple, extending the previous results of Borwein, Hare, Mossinghoff. We find examples of polynomials whose products and squares have no Littlewood or Newman multiples, while the original polynomials possess such multiples.

Described results were presented in the paper "On Littlewood and Newman polynomial multiples of Borwein Polynomials" (to appear in AMS *Mathematics of Computation*).

Gyula Katona, **An improvement on the intersecting shadow theorem**

Introduce the notation $[n]=\{1,2,\ldots,n\}$, then the family of all k-element subsets of $[n]$ can be denoted as $\binom{[n]}{k}$. Suppose that $\mathcal{F}\subset\binom{[n]}{k}$. Then its shadow $\sigma(\mathcal{F})$ is the family of all $k-1$-element sets obtained by deleting single elements from the members of $\mathcal{F}$, that is $\sigma(\mathcal{F})=\{A: |A|=k-1,\ \text{there is an } F\in\mathcal{F} \text{ such that } A\subset F\}$. The shadow theorem determines $\max|\sigma(\mathcal{F})|$ for fixed n,k and $|\mathcal{F}|$. We will survey variants of the shadow theorem. One of them (proved by the author in 1964) determines the minimum of the ratio $|\sigma(\mathcal{F})|/|\mathcal{F}|$ for fixed n and k. The main goal of the present talk is to exhibit a new improvement of this result for the case when $|\mathcal{F}|$ is large.

Sandor Kiss, **On the structure of sets which has coincide representation functions**

For a set of nonnegative integers S let $RS(n)$ denote the number of restricted representations of the integer n as the sum of two different terms from S. In this talk I partially describe the structure of the sets, which has coincide representation functions. This is joint work with Csaba Sándor.

Maxim A. Korolev, **On the estimates of Kloosterman sums**

Let $q\geq 3$, a,b be the integers, $(a,q)=1$. The complete Kloosterman sum modulo q is an exponential sum

$$S_q = \sum_{\substack{n=1\\(n,q)=1}}^{q} e_q(a\overline{n}+bn),$$

where, as usual, $\overline{n}$ denotes the inverse residue, that is, the solution of the congruence

$$n\overline{n} \equiv 1 \pmod q$$

and $e_q(u)=e^{2\pi iu}$. An incomplete Kloosterman sum is the sum of the type

$$S_q(N) = \sum_{\substack{n=1\\(n,q)=1}}^{N} e_q(a\overline{n}+bn),$$

where $1<N<q$.

The classic result of A. Weyl (1948) implies the non-trivial bounds for any incomplete sums $S_q(N)$ with length $N\geq q^{0.5+\varepsilon}$, where ε is any fixed positive constant. First non-trivial bounds for incomplete sums for general moduli q were obtained by A.A. Karatsuba in 1990'th by his ingenious and original method.

In the talk, we give a short survey of recent results concerning the estimates of incomplete Kloosterman sums and its analogues obtained by the speaker.

Antanas Laurinčikas, **Joint discrete universality of zeta-functions of certain cusp forms**

Let $F(z)$ be a normalized Hecke eigen cusp form of weight κ for the full modular group with the Fourier series expansion

$$F(z)=\sum_{m=1}^{\infty} c(m)e^{2\pi imz}, \qquad c(1)=1.$$

The associated zeta function $\zeta(s,F)$, $s=\sigma+it$, is defined, for $\sigma>\frac{\kappa+1}{2}$, by the Dirichlet series

$$\zeta(s,F)=\sum_{m=1}^{\infty}\frac{c(n)}{m^s},$$

and is analytically continued to an entire function.

In [1], it was proved that the function $\zeta(s,F)$ is universal in the Voronin sense, i.e., if K is a compact subset of the strip $\left\{s\in\mathbb{C}:\ \frac{\kappa}{2}<\sigma<\frac{\kappa+1}{2}\right\}$ with connected complement, and $f(s)$ is a continuous non-vanishing function on K that is analytic in the interior of K, then, for every $\varepsilon>0$,

$$\liminf_{T\to\infty}\frac{1}{T}\text{meas}\left\{\tau\in[0,T]:\sup_{s\in K}|\zeta(s+i\tau,F)-f(s)|<\varepsilon\right\}>0.$$

A discrete version of the above theorem was obtained in [2].

The aim of this report is a joint discrete universality theorem for zeta-functions of normalized Hecke eigen cusp forms. For $j=1,\dots,r$, Let $F_j(z)$ be a normalized Hecke eigen cusp form of weight κ_j with Fourier coefficients $c_j(m)$, and let $\zeta(s,F_j)$ denote the corresponding zeta-function. Moreover, let $D_j=\left\{s\in\mathbb{C}:\ \frac{\kappa_j}{2}<\sigma<\frac{\kappa_j+1}{2}\right\}$, $\mathcal{K}_j$ be the class of compact subsets of the strip D_j with connected complements, and $H_0(K_j)$, $K_j\in\mathcal{K}_j$, denote the class of continuous non-vanishing functions on K_j that are analytic in the interior of K_j. For positive h_j, define

$$L(\mathbb{P},h_1,\dots,h_r,\pi)=\left\{(h_1\log p:p\in\mathbb{P}),\dots,(h_r\log p:p\in\mathbb{P}),\pi\right\},$$

where $\mathbb{P}$ is the set of all prime numbers. Then we have the following joint discrete universality theorem.

Theorem 1. *Suppose that the set $L(\mathbb{P},h_1,\dots,h_r,\pi)$ is linearly independent over $\mathbb{Q}$. For $j=1,\dots,r$, let $K_j\in\mathcal{K}_j$ and $f_j(s)\in H_0(K_j)$. Then, for every $\varepsilon>0$,*

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\leqslant k\leqslant N:\sup_{1\leqslant j\leqslant r}\sup_{s\in K_j}|\zeta(s+ikh_j,F_j)-f(s)|<\varepsilon\right\}>0.$$

References

[1] A. Laurinčikas and K. Matsumoto, The universality of zeta functions attached to a certain cusp forms, *Acta Arith.* **98** (2001), 345–359.

[2] A. Laurinčikas, K. Matsumoto and J. Steuding, Discrete universality of L-functions of new forms. II, *Lith. Math. J.* **56** (2016), 207–218.

Eugenijus Manstavičius, **On the order statistics of component sizes of a random combinatorial structure**

We attempt to soften the conditions under which the component vector of a large random decomposable structure obeys the Poisson-Dirichlet distribution. The result confined to permutations sampled from the symmetric group $\mathbf{S}_n$ according to the generalized Ewens probability measure $\nu_{n,\theta}$ sounds as follows. Define the measure by ascribing a probability

$$\nu_{n,\Theta}(\{\sigma\}):=\frac{1}{n!m(n)}\prod_{j\le n}\theta_j^{k_j(\sigma)},\qquad m(n):=\sum_{1s_1+\cdots+ns_n=n}\prod_{j=1}^{n}\left(\frac{\theta_j}{j}\right)^{s_j}\frac{1}{s_j!},$$

for each $\sigma \in \mathbf{S}_n$ having $k_j(\sigma) \geq 0$ cycles of length j. Here $\theta_j \geq 0$ are arbitrary weights and $s_j \in \mathbf{N}_0$ if $j \in \mathbf{N}$. Let $L_1(\sigma) \geq L_2(\sigma) \geq \cdots L_w(\sigma) \geq 1$ be all the lengths of cycles in σ listed according to their multiplicities, $Y_n = n^{-1}\big(L_1, L_2, \ldots, L_w, 0, \ldots\big)$, where $L_i := L_i(\sigma)$, and $P_{n,\Theta} = \nu_{n,\Theta} \cdot Y_n^{-1}$ be the distribution of the latter.

Theorem. *If $\theta_j \leq C < \infty$, $j \geq 1$,*

$$\frac{1}{n}\sum_{j\leq n} \theta_j \to \theta > 0$$

and $n \to \infty$, then $P_{n,\Theta}$ weakly converges to the Poisson-Dirichlet distribution $PD(\theta)$.

Thus, the traditionally used Logarithmic Condition $\theta_j \to \theta$ as $j \to \infty$ is relaxed. The extension already covers the so-called A-permutations, that is, permutations whose cycles lengths belong to a set $A \subset N$ having the asymptotic density θ. The result is invariant under the shift $\theta_j \mapsto \rho^j \theta_j$ with an arbitrary constant $\rho > 0$. The analytic ingredient of the proof is the fact that, under the given conditions, $m(n)$ is regularly varying at infinity sequence.

An historical account of the problem can be found in Section 5.7 of the monograph by R. Arratia, A.D. Barbour and S. Tavaré (*Logarithmic Combinatorial Structures: A Probabilistic Approach*, EMS Publishing House, Zürich, 2003).

Antoine Marnat, **Transference between homogeneous and inhomogeneous Diophantine approximation on manifolds**

We provide an extension of the transference results of Beresnevich and Velani connecting homogeneous and inhomogeneous Diophantine approximation on manifolds and provide bounds for inhomogeneous Diophantine exponents of affine subspaces and their nondegenerate sub manifolds.

Mate Matolcsi, **Colouring the plane with sets avoiding the unit distance**

How many colours are needed to colour the plane such that the endpoints of each unit segment have different colours? This is a classical question for which the answer is known to be between 4 and 7. However, if the colour classes are assumed to be measurable then Falconer proved that at least 5 colours are needed. We outline a new proof of this fact based on a Delsarte-type Fourier analytic argument.

Dezső Miklós, **The vertex and edge sign balance of (hyper)graphs**

This is joint work with Justin Ahmann, Elizabeth Collins-Wildman, John Wallace, Shun Yang,Yicong Guo, Gyula Y. Katona Amanda Burcroff, Haochen Li, Greg McGrath.

Pokrovskiy and, independently, Alon, Huang and Sudakov introduced the MMS (Manickam-Miklós-Singhi) property of hypergraphs: "for every assignment of weights to its vertices with nonnegative overall sum, the number of edges whose total weight is nonnegative is at least the minimum degree of H". This leads to the definition of the following hypergraph parameter: The vertex sign balance of a hypergraph is the minimum number of edges whose total weight is nonnegative, where the minimum is taken over all assigments of weights to the vertices with nonnegative overall sum.

The vertex sign balance is always between 0 and the minimum degree of the graph or hypergraph, both bounds being sharp. General and special properties (for graphs or ypergraphs) of this parameter will be presented. I particular,it will be shown that this parameter measures the "matchingability" of graphs and similar observation can be made for hyperhraphs. This characterization of the vertex sign balance of the graphs leads to the result that the question if a graph or hypergraph has the MMS property is NP-complete.

The dual of the vertex sign balance, the edge sign balance is defined by the minimum number of vertices whose total weight is nonnegative, where the minimum is taken over all assigments of weights to the edges with nonnegative overall sum and the weight of a vertex is obtained as the sum of the weights of the edges containing the vertex. For graphs this parameter is always between 1 and 2, being two iff the graph is bipartite. However, for 3-uniform graphs, while being three-partite would yield the edge sign balance being three, the opposite is probably not true. The edge sign balance can be a "measure" for "coverability",ore accessible than three- (or multi-) partitness.

Nikolay G. Moshchevitin, **On some open problems in Diophantine Approximation and related fields**

We suppose to discuss several unsolved problems such as Zaremba's conjecture, related problems about numbers with missed digits, discrepancy bounds for normal numbers, Peres-Schlag's method in Diophantie approximation and some conjectures in multidimensional Diophantine approximation.

Brendan Murphy, **Rich lines in general position**

n a Cartesian product point set $Y \times Y$ (a "grid") if it contains at least $\alpha|Y|$ points of $Y \times Y$. Bounds for the number of lines that are rich in a grid are related to sum-product estimates.

Solymosi conjectured that the number of lines that are α-rich in a grid and in general position, meaning that no two are parallel and no three intersect in a common point, must be bounded by a constant depending on α. We prove upper bounds for the number of rich lines in general position, and we prove that these estimates are fairly sharp. In particular, the number of rich lines in general position of an $N \times N$ grid tends to infinity with N, disproving Solymosi's conjecture.

The upper bounds are proved using a group action version of the asymmetric Balog-Szemerédi-Gowers lemma. The lower bounds use techniques from the theory of expander graphs. We will briefly discuss these connections as time permits.

Peter Pal Pach, **On progression-free sets via the polynomial method**

In this talk we will look at a new variant of the polynomial method which was first used to show that sets avoiding 3-term arithmetic progressions in groups like $\mathbb{Z}_4^n$ and $\mathbb{F}_q^n$ are exponentially small (compared to the size of the group). Since then many interesting applications of this method were shown.

Fedor Petrov, **Group algebra and the structure of product set**

A recent breakthrough by E. Croot, V. Lev and P. Pach led to a number of results in arithmetic combinatorics of groups which are direct products of small cyclic groups, like the vector spaces over a finite field. We present a group algebras (a-la Olson) version of their method, which turns out to work for a wide class of groups, including not commutative like upper triangular matrices.

Roman Prosanov, **Chromatic numbers of spaces and fractional covering technique**

The chromatic number of a metric space is the smallest number of colors sufficient for coloring all points of this space in such a way that any two points at some fixed distance d have different colors. By $\chi(\mathbf{R}^n)$ denote the chromatic number of the Euclidean space $\mathbf{R}^n$ with forbidden distance 1 (in the Euclidean case the chromatic number does not depend on a value of forbidden distance). We also discuss the spherical case. We consider the sphere embedded in the Euclidean space and measure distances in the Euclidean sense. To determine the chromatic numbers with different forbidden distances for the sphere with some fixed radius is equivalent to determine the chromatic numbers $\chi(S_R^n)$ with forbidden distance 1 for spheres with different radii $R \geq 1/2$.

The study of chromatic numbers has a long history. The exact value of the chromatic number is not known even in the case of $\mathbf{R}^2$. In the case of arbitrary n we have

$$(1.239 + o(1))^n \leq \chi(\mathbb{R}^n) \leq (3 + o(1))^n. \tag{1}$$

The lower bound is due to Raigorodskii [8] and the upper bound is due to Larman and Rogers [3].

In 1981 Erdős conjectured that for any fixed $R \geq 1/2$, $\chi(S_R^n)$ is growing as n tends to infinity. In 1983 this was proved by Lovász [4] using interesting mixture of combinatorial and topological techniques. Among other things, in this paper Lovász claimed that for $R < \sqrt{\frac{n+1}{2n+4}}$ (i.e. when the side of regular $(n+1)$-dimensional simplex inscribed in our sphere is less than 1) we have $\chi(S_R^n) = n + 1$. However, in 2012 Raigorodskii showed [9] that this statement was wrong. In [9] it was shown that actually for any fixed $R > 1/2$ the quantity $\chi(S_R^n)$ is growing exponentially.

It is clear that

$$\chi(S_R^n) \leq (3 + o(1))^n$$

because S_R^n is a subset of $\mathbf{R}^{n+1}$. Despite the remarkable interest to this problem in general there are no better bounds. For spheres of small radii ($R \leq 3/2$) the works of Rogers imply much stronger bound. Consider the spherical cap on S_R^n of such radius that the Euclidean diameter of this cap is less than 1. Then we cover S_R^n with copies of this cap and paint every cap in own color. In such a way, we get

$$\chi(S_R^n) \leq (2R + o(1))^n.$$

There is a strong connection between chromatic numbers and geometric covering problems. The recent work of M. Naszódi [5] provides a new approach to geometric coverings (partially crossing with the papers of Artstein-Avidan, Raz and Slomka, [1], [2]). It was suggested to consider some finite hypergraph in such a way that edge-covering of this hypergraph implies the desired geometric covering. We can study covering numbers of finite hypergraphs via the famous result of Lovász on the ratio between the optimal covering number and the optimal fractional covering number. It is wondering that the fractional covering number of our hypergraph can be finely estimated from geometric setting.

We apply this technique to the study of chromatic numbers. This allows us to give a new better bound in the spherical case and a new proof of the upper bound in (1). More precisely, define a function

$$x(R) = \begin{cases} \sqrt{5 - \frac{2}{R^2} + 4\sqrt{1 - \frac{5R^2-1}{4R^4}}}, & R \geq \frac{\sqrt{5}}{2} \\ 2R, & \frac{1}{2} \leq R \leq \frac{\sqrt{5}}{2} \end{cases}$$

Theorem 1. *For $R \geq \frac{1}{2}$ we have $\chi(S_R^n) \leq (x(R) + o(1))^n$.*

It is clear that the base of exponent is always less than 3. (However, we should mention that it tends to 3 as R tends to infinity.) Also, it can be shown that it is less than $2R$ at the interval $(\frac{\sqrt{5}}{2}; \frac{3}{2})$. So, we improve all current bounds except the interval $\frac{1}{2} \leq R \leq \frac{\sqrt{5}}{2}$. But our method provides another proof of the bound $\chi(S_R^n) \leq (2R + o(1))^n$ in this case.

Let $B_R^{n+1} \subset \mathbf{R}^{n+1}$ be a Euclidean ball of radius R (centered in the origin). By $\chi(B_R^{n+1})$ denote the chromatic number of B^{n+1} (with forbidden distance 1).

Theorem 2. *For $R \geq \frac{1}{2}$ we have $\chi(B_R^{n+1}) \leq (x(R) + o(1))^n$.*

In our talk we discuss these results (from [7]) and a new proof of the upper bound in (1) (from [6]).

References

[1] S. Artstein-Avidan, O. Raz, *Weighted covering numbers of convex sets*, Adv. Math. **227** (2011), no. 1, 730–744.

[2] S. Artstein-Avidan, B. A. Slomka, *On weighted covering numbers and the Levi-Hadwiger conjecture*, arXiv:1310.7892 [math].

[3] D.G. Larman, C.A. Rogers, *The realization of distances within sets in Euclidean space*, Mathematika, **19** (1972), 1–24.

[4] L. Lovász, *Self-dual polytopes and the chromatic number of distance graphs on the sphere*, Acta Sci. Math. **45** (1983), 317-323.

[5] M. Naszódi, *On some covering problems in geometry*, Proceedings of the American Mathematical Society, **144** (2016), 3555-3562.

[6] R.I. Prosanov, *A new proof of the Larman–Rogers upper bound for the chromatic number of the Euclidean space*, arXiv:1610.02846 [math].

[7] R.I. Prosanov, *Problems in Euclidean Ramsey theory*, Master's thesis, Lomonosov Moscow State University (2017).

[8] A.M. Raigorodskii, *On the chromatic number of a space*, Russian Mathematical Surveys, **55** (2000), no.2, 351–352.

[9] A.M. Raigorodskii, *On the chromatic numbers of spheres in* $\mathbf{R}^n$, Combinatorica, **32** (2012), no. 1, 111–123.

Radhakrishnan Nair, **Weyl's theorem on uniform distribution and ergodic theorems**

Suppose $(k_n)_{n\geq 1}$ is Hartman uniformly distributed and good universal. Also suppose ψ is a polynomial with at least one coefficient other than $\psi(0)$ an irrational number. We adapt an argument due to H. Furstenberg to prove that the sequence $(\psi(k_n))_{n\geq 1}$ is uniform distribution modulo one. This is used to give some new families of Poincaré recurrent sequences. In addition we show these sequences are also intersective and Glasner.

Joël Rivat, **On the digits of primes and squares**

We will give a survey of our results on the digits of primes and squares (joint works with Michael Drmota and Christian Mauduit).

Oliver Roche-Newton, **On the size of the set $AA + A$ over the reals**

In the spirit of the Erdős-Szemerédi sum-product conjecture, it is widely believed that sets defined by a combination of additive and multiplicative operations on a finite set A are significantly larger in size than the original set. One of the first questions of this type that arises concerns the size of the set $AA + A$. This talk will discuss some new bounds for this problem.

Anne de Roton, **Small sumsets in $\mathbb{R}$.**

We describe the structure of subsets A and B of real numbers such that the sumset $A+B$ has small measure. We first prove a continuous version of Freiman's $3k-4$ theorem as generalized by Grynkiewicz, this result gives some information on the structure of A, B and $A+B$ when $\lambda(A+B) < \lambda(A)+\lambda(B)+\min(\lambda(A), \lambda(B))$. We also use a result on small sumsets on the circle to describe sets A of real numbers such that $\lambda(A+A) < (3+c)\lambda(A)$ for a small constant c. This last result is joint work with Pablo Candela.

Misha Rudnev, **Counting distinct cross-ratios**

It appears that the question, what is the minimum number of distinct cross-ratios, determined by a finite set A in a field K, sufficiently small, if K has positive characteristic p, is deep. For distinct elements a, b, c, d of A, the cross-ratio is the fraction $(a-b)(c-d)/(a-c)(b-d)$. The talk discusses state of the art bounds, connections, and open questions.

Johannes Schleischitz, **Classical exponents of Diophantine approximation**

The talk aims to present recent results on classical exponents of Diophantine approximation. Let $n \geq 1$ denote an integer and ζ be a real number. In 1932, Mahler introduced the exponent $w_n(\zeta)$ as the supremum of η such that

$$0 < |a_0 + a_1\zeta + \cdots + a_n\zeta^n| \leq (\max_{0\leq j\leq n} |a_j|)^{-\eta}$$

has infinitely many integral solution vectors $(a_0, \ldots, a_n)$. In 2005, Bugeaud and Laurent defined exponents $\lambda_n(\zeta)$ concerning simultaneous rational approximation to $(\zeta, \zeta^2, \ldots, \zeta^n)$. Mahler partitioned the transcendental real numbers with respect to the growth of the sequence $(w_n(\zeta))_{n\geq 1}$ in S-numbers, T-numbers and U-numbers. A recent result presented in the talk is that, rather surprisingly, the Mahler classification is induced as well by imposing some natural decay porperties on the sequence of exponents $(\lambda_n(\zeta))_{n\geq 1}$. The method has several applications. For example, metric results on the Hausdorff dimension of the sets $\{\zeta \in \mathbb{R} : \lambda_n(\zeta) \geq a\}$, corresponding to the set of vectors on the Veronese curve approximable to a given degree $a > 1/n$ by rational vectors, are deduced.

Tomasz Schoen, **New upper bounds for additive Ramsey numbers**

A classic result of Rado states that for every homogenous regular equation with integer coefficients there is the least natural number $R(n)$ such that if the elements of $[N] = \{1, \ldots, N\}$ are colored into n colors for $N > R(n)$, then there is a monochromatic solution to the equation. While density results provide quite accurate bounds for $R(n)$ in case of invariant equations (i.e. the sum of coefficients equals zero), the general upper bounds known have been tower-like in nature. We present new upper bound $R(n) = \exp(O(n^4 \log^4 n))$ for all homogenous regular equations. On the other hand it is easy to observe that for every non-invariant equation we have $R(n) \geq \exp(cn)$, where $c > 0$ depends only on the coefficients of the equation. We also show new upper bounds for the Schur-like numbers for the equation $x_1 + x_2 + x_3 = y_1 + y_2$ and for the van der Waerden numbers $W(3; k)$. This is a joint work with **Karol Cwalina**.

Alisa Sedunova, **A logarithmic improvement in the Bombieri-Vinogradov theorem**

We improve the best known to date result of Dress-Iwaniec-Tenenbaum, getting $(\log x)^2$ instead of $(\log x)^{\frac{5}{2}}$. We use a weighted form of Vaughan's identity, allowing a smooth truncation inside the procedure, and an estimate of Graham based on the work due to Barban-Vehov related to Selberg's sieve. We give effective and non-effective versions of the main result.

Aliaksei Semchankau, **Maximal subsets avoiding arithmetic progressions in sets**

Let B be a set of integers, and $k \geq 3$. We define a function $f_k(B)$ to be the size of a maximum subset $A \subset B$ which does not contain arithmetical progressions of length k. Intuitively, f_k characterizes the arithmetical structure of B, namely, if f_k is small, then B seems to be structured. It is natural to suppose that $f_k(B)$ attains its minimal value in the case when B is close to an arithmetical progression. Formally, is it true that $\min_{|B|=n} f_k(B) \sim g_k(n) := f_k(\{1, 2, \cdots n\})$? In 1975 J. Komlos, M. Sulyok, E. Szemerédi proved a very general result about sets avoiding several linear equations, and they obtained, in particular, that $f_3(n) \geq (c + o(1))\, g_3(n)$, where c is $1/2^{15}$. Here we prove that for every k an inequality $f_k(n) \geq (1/4 + o(1))\, g_k(n)$ holds for infinitely many n. Our main tool is the following hypothesis which is an interesting in its own right.

Definition: A set $Y = \{y_1, y_2, \cdots, y_n\}$ is called a *compression* of a set $X = \{x_1, x_2, \cdots, x_n\}$ if for any i, j, k such that $x_i - 2x_j + x_k = 0$ it follows that $y_i - 2y_j + y_k = 0$. In other words, it means that Y contains the same progressions as X.

Hypothesis. For any $1 > \epsilon > 0$ there exists a function $h_\epsilon(n)$ which tends to infinity slower than any power with the next property: for any set X of size n one can delete at most ϵn elements such that the remaining set can be compressed into a subset of the segment $\{1, 2, \cdots, nh_\epsilon(n)\}$.

This hypothesis has been proved in a particular case when $\epsilon > 3/4$. In this case $h_\epsilon(n)$ can be taken as $h_\epsilon(n) = C_\epsilon \log n$, where C_ϵ is a constant which depends on ϵ only.

Alexander Semenov, **On the j-chromatic number of random hypergraphs**

Let $H = (V, E)$ be a hypergraph. A hypergraph H is k-uniform if all edges of H are of size k. A random k-uniform hypergraph $H(n, k, p)$ is an k-uniform hypergraph on n labeled vertices $V = \{v_1, \dots, v_n\}$, in which every subset $e \subset V$ of size k is chosen to be an edge of H randomly and independently with probability p, where p may depend on n. We will study the chromatic number of random hypergraphs. Actually, a family of chromatic numbers can be defined.

Definition 1. *For an integer j, a j-independent set in a hypergraph $H(V, E)$ is a subset $W \subset V$ such that for every edge $e \in E : |e \cap W| \leqslant j$.*

Definition 2. *A j-coloring of $H(V, E)$ is a partition of the vertex set V of H into j-independent sets, so called colors. The j-chromatic number $\chi_j(H)$ of H is the minimal number of colors in a j-coloring of H.*

The main interest of this work is the asymptotic behavior of the property of hypergraph $H(n, k, p)$ to have its j-chromatic number equal to 2. By asymptotic properties of $H(n, k, p)$ we consider n as tending to infinity while k is kept constant.

It can be showed that the previously mentioned property of random hypergraph has a sharp threshold [1]. The case of $j = k - 1$ was intensively studied and authors of [2] have found the upper and lower bound for that threshold but there was a large gap between those bounds. Later in works [3], [4] and [5] bounds were improved and the gap was reduced to the $O_k(1)$.

Here we consider the generalization to the case when j is less than k by some constant value. Main result is showed in a theorem below.

Theorem 2. *Let $d > 1$ be a constant integer. There exists $k_0 \in \mathbb{N}$ such that for each $k > k_0$, $j = k - d$ and*

$$c < \frac{2^{k-1} \ln 2}{\sum\limits_{i=j+1}^{k} \binom{k}{i}} - \frac{\ln 2}{2} + O(k^{1-d}) \tag{1}$$

the j-chromatic number $\chi_j(H(n,k,cn/\binom{n}{k}))$ is equal to 2 with high probability as $n \to \infty$. Otherwise if

$$c > \frac{2^{k-1}\ln 2}{\sum\limits_{i=j+1}^{k}\binom{k}{i}} - \frac{\ln 2}{2} + O\left(\left(\frac{3}{4}\right)^k\right), \tag{2}$$

then j-chromatic number $\chi_j(H(n,k,cn/\binom{n}{k}))$ is greater than 2 with high probability as $n \to \infty$.

As reader can see, in comparison with the case $j = k-1$ the gap between upper bound (2) and lower bound (1) in the theorem tends to zero with growth of k.

References

[1] H. Hatami, M. Molloy, "Sharp thresholds for constraint satisfaction problems and homomorphisms", *Random Structures and Algorithms, 33:3 (2008)*, 310-332.

[2] N. Alon, J. Spencer, "A note on coloring random k-sets", Unpublished manuscript.

[3] D. Achlioptas, J.H. Kim, M. Krivelevich, P. Tetali, "Two-colorings random hypergraphs", *Random Structures and Algorithms, 20:2 (2002)*, 249-259.

[4] D. Achlioptas, C. Moore, "Random k-SAT: two moments suffice to cross a sharp threshold", *SIAM Journal on Computing, 36:3 (2005)*, 740-762.

[5] A. Coja-Oghlan, L. Zdeborova, "The condensation transition in random hypergraph 2-coloring", *Proc. 23rd Annual ACM-SIAM Symposium on Discrete Algorithms, SIAM, 2012*, 241-250.

Dmitry Shabanov, The Erdős–Hajnal problem on colorings of hypergraphs, ts on-line generalizations and related questions

The talk deals with the classical problem concerning colorings of hypergraph that was stated by Erdős and Hajnal in the 60-s. The problem is to find the minimum possible number of edges in an n-uniform hypergraph with chromatic number greater than two. We will survey recent progress in this question and also discuss different generalizations. One of them is concerned with on-line and list on-line colorings. On-line coloring of a hypergraph is a game with two players, Lister and Painter, in which Lister picks a vertex one by one (or a set of vertices) and Painter should choose a color for the given vertex (or choose a subset to be colored). The problem is to find an extremal value of some characteristics which admits a winning strategy for Painter. We establish the asymptotic behavior of the list on-line chromatic number for the complete multipartite graphs and hypergraphs. We will also give some results for the Erdős–Hajnal–type problems for on-line colorings. This is joint work with Alina Khuzieva and Polina Svyatokum.

Talia Shaikheeva, List chromatic numbers of the complete multi-partite hypergraphs

This is joint work with Dmitrii Shabanov.

The paper deals with list colorings of uniform hypergraphs. We start with recalling some definitions.

Let $H = (V, E)$ be a hypergraph. A *vertex coloring* f is a mapping from the vertex set V to some set of colors C. A coloring is called *proper* for H if there is no monochromatic edges in E under it, i.e. formally, for every edge $A \in E$, $|\{f(v) : v \in A\}| > 1$. *The chromatic number* of H, $\chi(H)$, is the minimum s such that there is proper coloring for H with s colors.

We study the extension of the chromatic number, which is called the list chromatic number. A hypergraph $H = (V, E)$ is said to be *s-choosable* if for every family of sets $L = \{L(v) : v \in V\}$, *the list assignment*, such that $|L(v)| = s$ for any $v \in V$, there exists a proper coloring from the lists, i.e. for every $v \in V$, we should use a color from $L(v)$. The *list chromatic number* of H, denoted by $\chi_l(H)$, is the minimum s such that H is s-choosable.

Let $H(m, r, k)$ denote the complete r-partite k-uniform hypergraph with m vertices in every part, in which any edge takes exactly one vertex from some $k \leqslant r$ parts. The main result of the paper provides the asymptotic behavior for the list chromatic number of $H(m, r, k)$.

Theorem. *For fixed* $2 \leqslant k \leqslant r$*,*

$$\chi_l(H(m, r, k)) = (1 + o(1)) \log_{\frac{r}{r-k+1}}(m) \text{ as } m \to \infty.$$

The same asymptotic representation holds for any functions $r = r(m)$*,* $k = k(m)$*, such that* $\ln r = o(\ln m)$*.*

The proof was obtained by using the connection between list colorings of uniform hypergraphs and multiple coverings by independent sets.

References

[1] D.A. Shabanov, "On a generalization of Rubin's theorem", *Journal of Graph Theory*, **67**:3 (2011), 226–234.

[2] P. Haxell, J. Verstraëte, "List coloring hypergraphs", *Electronic Journal of Combinatorics*, **17** (2010), Research paper N 129.

[3] A. V. Kostochka, "On a theorem by Erdős, Rubin and Taylor on choosability of complete bipartite graphs", *Electronic Journal of Combinatorics*, **9** (2002).

Iekata Shiokawa, **Irrationality exponents of numbers related with e Cahen's constant**

We give lower and upper bound for the irrationality exponents $\mu(\alpha)$ of continued fractions

$$\alpha = \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{\ddots}}}},$$

where $a_n, b_n \in \mathbb{Z}$ satisfy certain conditions. As an application we prove the following. Let

$$C_l = \sum_{n=0}^{\infty} \frac{(-1)^n}{(S_n - 1)^l} \qquad (l = 1, 2, 3, \ldots),$$

where $\{S_n\}$ is Sylvester's sequence defined by $S_0 = 2$, $S_{n+1} = S_n^2 - S_n + 1$ $(n \geq 0)$. Cahen's constant $C = C_1$ is known to be transcendental. We prove that $\mu(C_1) = 3$ and

$$2 + \frac{2}{3l - 2} \leq \mu(C_l) \leq 2 + \frac{6(l-1)}{3l+1} \qquad (l = 2, 3, 4, \ldots).$$

Ilya Shkredov, **On the asymmetric sum-product phenomenon**

Using some new observations connected to higher energies, we obtain quantitative lower bounds on $\max\{|AB|, |A + C|\}$ and $\max\{|(A + \alpha)B|, |A + C|\}$, $\alpha \neq 0$ in the regime when the sizes of finite subsets A, B, C of a field differ significantly.

Darius Šiaučiūnas and **Mindaugas Stoncelis**, **On weighted discrete universality of periodic zeta-functions**

The periodic zeta-function $\zeta(s; \mathfrak{a})$, $s = \sigma + it$, where $\mathfrak{a} = \{a_m : m \in \mathbb{N}\}$ is a periodic sequence of complex numbers, in the half plane $\sigma > 1$ is defined by the series

$$\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s}.$$

Moreover, $\zeta(s; \mathfrak{a})$ can be meromorphically continued to the whole complex plane.

Universality of periodic zeta-functions was considered by various authors: Bagchi, Steuding, Kaczorowski, Laurinčikas, Stoncelis, Šiaučiūnas. Our report is devoted to a weighted discrete universality theorem for the function $\zeta(s; \mathfrak{a})$. Its statement include the weight function $w(u)$, $u \geqslant 1$. Let

$$V_N = \sum_{k=1}^{N} w(k).$$

We suppose that $\lim_{N \to \infty} V_N = +\infty$, moreover, the function $w(u)$ has a continuous derivative satisfying the estimate

$$\int_1^N u|w'(u)| \mathrm{d}u \ll V_N.$$

As usual, let $\mathcal{K}$ be the class of compact subsets of the strip $\left\{s \in \mathbb{C}: \frac{1}{2} < \sigma < 1\right\}$ with connected complements, and $H_0(K)$, $K \in \mathcal{K}$, be the class of continuous non-vanishing functions on K that are analytic in the interior of K. Moreover, let I_A denote the indicator function of the set $A \subset \mathbb{R}$. Then the following theorem is true.

Theorem. *Suppose that the function $w(u)$ satisfies the above hypotheses, the sequence $\mathfrak{a}$ is multiplicative, and that α, $0 < \alpha < 1$, and $h > 0$ are fixed. Let $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{N \to \infty} \frac{1}{V_N} \sum_{k=1}^{N} w(k) I_{\left\{k: \sup\limits_{s \in K} |\zeta(s+ik^\alpha h; \mathfrak{a}) - f(s)| < \varepsilon\right\}}(k) > 0.$$

A continuous version of this Theorem was obtained in [1].

References

[1] R. Macaitienė, M. Stoncelis and D. Šiaučiūnas, A weighted universality theorem for periodic zeta-functions, *Math. Modell. Analysis* **22**(1) (2017), 95–105.

Raivydas Šimenas, **Zero free regions of the Lerch zeta-function**

One of the most researched functions in all of mathematics is the Riemann zeta-function. The Lerch zeta-function is a generalization of it. Like the Riemann zeta-function, the Lerch zeta-function is defined as a Dirichlet series. In our talk, we show that similarly to the Riemann zeta-function, the Lerch zeta-function has zero free regions in the right and the left parts of the complex plane excluding a strip around the critical region and around a line which goes from around the origin to the upper left part of the complex plane.

Jozsef Solymosi, **Sum-product bounds for complex matrices.**

Improving earlier results with Vu and with Tao, we show a new sum-product bound for a family of matrices. The proof also implies the bound of Konyagin and Rudnev for complex numbers. For an n-element set of k by k matrices, A, we show $|A+A| + |AA| > c|A|^{4/3}/log|A|$, where c depends on k only.

Gediminas Stepanauskas, **Jonas Šiaulys**, **Limit distributions for some sets of additive functions**

Let $x > 2$ and $f_x : \mathbb{N} \to \mathbb{R}$ be a set of strongly additive prime indicators. The limit behaviour of distributions

$$\frac{1}{[x]} \sum_{\substack{n \leqslant x \\ f(n) - \alpha(x) < u\beta(x)}} 1$$

was considered in the probabilistic number theory very often with different centering and normalizing functions $\alpha(x)$ and $\beta(x)$. In the books [1], [2], [3], and works cited there, one can find almost all classical results and their historical context.

An object of the talk is a weak convergence of distributions

$$\frac{1}{[x]} \sum_{\substack{n \leqslant x \\ f_x(n) < u}} 1, \tag{1}$$

$$\frac{1}{[x]} \sum_{\substack{n \leqslant x \\ f_{1x}(A(n)) + f_{2x}(B(n)) < u}} 1, \tag{2}$$

where $A(n), B(n)$ are two arithmetically interesting subsequences of positive integers. Some distributions can arise as limit distributions for (1), as $x \to \infty$, some ones can not. From the limit behaviour of (2), interesting conclusions on the common asymptotic properties and multiplicative structure of the sets $A(n), B(n)$ can be deduced. For instance,

$$\#\left\{p : \begin{array}{c} p \text{ prime}, p \leqslant x, p+1 \text{ and } p+2 \\ \text{have no prime factors in } (\log x, (\log x)^2) \end{array}\right\} \sim \frac{x}{4 \log x}$$

as $x \to \infty$.

References

1. P.D.T.A. Elliott, *Probabilistic Number Theory, I*, Springer-Verlag, New York, 1979.
2. P.D.T.A. Elliott, *Probabilistic Number Theory, II*, Springer-Verlag, New York, 1980.
3. J. Kubilius, *Probabilistic Methods in the Theory of Numbers* , Providence, Amer. Math. Soc. Translations of Math. Monographs, No **11** (1964).

Sophie Stevens, **An incidence theorem in finite fields and applications**

I will talk about an Szemerédi-Trotter type theorem in finite fields, and how this can be used as a tool in additive combinatorics. I will present a new, stronger incidence bound which is based on the geometry of the problem. I will then give some of the applications of this incidence bound, particularly to the area of sum-product theory. This is joint work with Frank de Zeeuw.

Leonhard Summerer, **Simultaneous approximation and associated spectra**

This talk will be devoted to explain how the parametric geometry of numbers leads to the definition of approximation spectra for the simultaneous approximation of real numbers through the successive minima functions with respect to a suitable lattice and a one parametric family of convex bodies. Some known spectra will be presented, as well as a general result due to Roy asserting that suitably defined spectra are compact, connected sets. Finally, in the context of the 3-dimensional spectrum, a geometrical interpretation for the lower and upper bounds of the second successive minimum function will be given.

Yohei Tachiya, **On algebraic relations between the values of the theta function**

In this talk, we show some algebraic relations between the values of the theta function $\theta(\tau) = \sum_{n=-\infty}^{\infty} e^{\pi i n^2 \tau}$, which is defined for τ in the upper half-plane $\mathbb{H}$. For example, the three values $\theta(\ell\tau)$, $\theta(m\tau)$, and $\theta(n\tau)$ are algebraically dependent over $\mathbb{Q}$ for any $\tau \in \mathbb{H}$, where $\ell, m, n \geq 1$ are integers. The algebraic independence results will be also presented. This is a joint work with Carsten Elsner and Florian Luca.

Katarzyna Taczała, **The degree of regularity of the equation $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i + b$**

A linear equation is called r-regular if for every r-coloring of $\mathbb{N}$ there exists a monochromatic solution to this equation. Kleitman and Fox showed that the equation $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i + b$ is $2n$-regular for any positive integer b and conjectured that this bound is tight. We will show that this conjecture is indeed true using a generalization of a structural theorem of Eberhard, Green and Manners on the sets with doubling less than 4.

This is joint work with Tomasz Schoen.

Rokas Tamosiūnas, **Symmetry of zeros of Lerch zeta-function for equal parameters**

That is a joint work with Ramūnas Garunkštis.

For most parameters, Lerch zeta-function zeros are distributed very chaotically, but there is a special case of equal parameters which behaves differently. Using calculations, visualizations and proofs we will show that nontrivial zeros either lie extremely close to the critical line or are distributed almost symmetrically with respect to the critical line.

Lola Thompson, **Divisor-sum fibers**

Let $s(\cdot)$ denote the sum-of-proper-divisors function, that is, $s(n) = \sum_{d|n,\ d<n} d$. Erdős–Granville–Pomerance–Spiro conjectured that, for any set $\mathcal{A}$ of asymptotic density zero, the preimage set $s^{-1}(\mathcal{A})$ also has density zero. We prove a weak form of this conjecture. In particular, we show that the EGPS conjecture holds for infinite sets with counting function $O(x^{\frac{1}{2}+\epsilon(x)})$. We also disprove a hypothesis from the same paper of EGPS by showing that for any positive numbers α and ϵ, there are integers n with arbitrarily many s-preimages lying between $\alpha(1-\epsilon)n$ and $\alpha(1+\epsilon)n$. If time permits, we will make some remarks on how the techniques used in the aforementioned work can be applied to obtain results on the number of solutions n to congruences of the form $\sigma(n) \equiv a \pmod{n}$. This talk is based on joint work with Paul Pollack and Carl Pomerance.

Jörg Thuswaldner, **Discrepancy bounds for $\boldsymbol{\beta}$-adic Halton sequences**

Van der Corput and Halton sequences are well-known low-discrepancy sequences. In the 1990ies Ninomiya defined analogues of van der Corput sequences for β-numeration and proved that they also form low-discrepancy sequences provided that β is a Pisot number. Hofer, Iacò, and Tichy define $\boldsymbol{\beta}$-adic Halton sequences and show that they are equidistributed for certain parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_s)$.

In this talk we give discrepancy estimates for $\boldsymbol{\beta}$-adic Halton sequences for which the components β_i are m-bonacci numbers. Our methods include dynamical and geometric properties of Rauzy fractals that allow to relate $\boldsymbol{\beta}$-adic Halton sequences to rotations on high dimensional tori. The discrepancies of these rotations can then be estimated by classical methods relying on W. M. Schmidt's Subspace Theorem.

Carlo Viola, **The multidimensional saddle-point method in Diophantine approximation**

For any positive integer N, the complex Morse lemma yields an asymptotic series expansion for an integral

$$I(\tau) = \int_\Gamma e^{\tau\, h(z_1,\ldots,z_N)} g(z_1,\ldots,z_N)\, \mathrm{d}z_1 \cdots \mathrm{d}z_N \qquad (\tau \to +\infty), \tag{$*$}$$

where Γ is a manifold in $\mathbb{C}^N$ and where g and h are holomorphic functions in an open subset of $\mathbb{C}^N$ containing Γ, provided Γ contains a nondegenerate saddle-point $\big(z_1^{(0)},\ldots,z_N^{(0)}\big)$ of $e^{h(z_1,\ldots,z_N)}$ at which $\operatorname{Re} h(z_1,\ldots,z_N)$ is maximal. Asymptotic formulae for N-dimensional integrals $I(\tau)$ as in $(*)$ have several applications to complex analysis and number theory. However, the main difficulty to apply the saddle-point method to the asymptotic study of $(*)$ is to locate the relevant saddle-point $\big(z_1^{(0)},\ldots,z_N^{(0)}\big)$, and to prove that the integration manifold Γ can be deformed to a manifold Λ, equivalent to Γ by Poincaré's theorem and containing the saddle-point as required. In my talk I will explain under which assumptions the deformation of Γ into Λ can be ensured, and I will sketch out some applications of asymptotic expansions of $(*)$ to Diophantine approximation.

Alexey Volostnov, **Sums of multiplicative characters with additive convolutions**

We obtain new estimates for binary and ternary sums of multiplicative characters with additive convolutions of characteristic functions of sets with small additive doubling. In particular, we improve a result of Mei-Chu Chang. The proof uses the Croot-Sisask almost periodicity lemma.

Dmitrii Zhelezov, **Convex sets can have thin additive bases**

An ordered set of real numbers is called convex if the gaps between two consecutive numbers in the set are strictly increasing. Convex sets play an important role in arithmetic combinatorics and often behave like multiplicatively structured sets (e. g. geometric progressions). In certain cases, however, convex sets may exhibit a rather unexpected behaviour. In the talk I will give such an example by constructing convex sets which admit a very thin additive basis, thus answering in the negative a question asked by Hegarty back in 2012. This is a joint work with I. Ruzsa.

Victoria Zhuravleva, **Periodic sequences modulo 1 and Pisot numbers**

The main problem of this talk is how to construct a periodic sequence modulo 1 which period has a given length using coefficients of a reccurence relation of a sequence corresponding to a minimal polynomial of a Pisot number α. If this sequence doesn't contain zero then we obtain a nontrivial lower bound for $\sup_{\xi\in\mathbb{R}} \liminf_{n\to\infty} ||\xi\alpha^n||$. In this talk I will discuss several constructions of sequences with periods of length 1, 2, 3 or 4.