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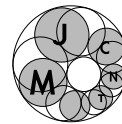
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## CORRIGENDUM

# Corrigendum to “On the maximum number of consecutive integers on which a character is constant”

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Theorem 1 in [1] should be corrected to the following:

**THEOREM 1.** *If  $\chi$  is any non-principal Dirichlet character to the prime modulus  $p$  which is constant on  $(N, N + H]$ , then*

$$H < \left\{ \frac{\pi}{2} \sqrt{\frac{e}{3}} + o(1) \right\} p^{1/4} \log p,$$

where the  $o(1)$  terms depends only on  $p$ . Furthermore,

$$H \leq \begin{cases} 3.38p^{1/4} \log p, & \text{for all odd } p, \\ 1.55p^{1/4} \log p, & \text{for } p \geq 10^{13}. \end{cases}$$

There are two differences:

1. The explicit constant for all  $p$  is changed from 3.64 to 3.38 (an improvement).
2. The bound  $1.55p^{1/4} \log p$  is proven for  $p \geq 10^{13}$  instead of  $p \geq 2.5 \cdot 10^9$ .

To prove that  $H(p) \leq 1.55p^{1/4} \log p$ , the only changes in the proof involve correcting Table 2 in [1]. To correct it, replace the first three rows of Table 2 of [1] with the following two rows:

$w$	$h$	$p$	$w$	$h$	$p$	$w$	$h$	$p$
8	36	$[10^{13}, 10^{13.36}]$	8	40	$[10^{13.36}, 10^{13.5}]$	8	41	$[10^{13.5}, 10^{14.4}]$
8	44	$[10^{14.4}, 10^{14.9}]$	9	45	$[10^{14.9}, 10^{16}]$	9	51	$[10^{16}, 10^{17}]$

**Table 1**

These two rows would replace the first three rows of Table 2 in [1]. The mistake in [1] stemmed from coding incorrectly the function  $\gamma(p, w, h)$ .

With respect to the error involving the bound for all  $p$ . In [1] we chose “large”  $h$  to circumvent the constraint  $(h/2)^{2/3}p^{1/3} \geq H$ , however, this constraint is illusory. If  $H$  is larger, we can pick a  $H' \leq H$  such that  $H' \leq (h/2)^{2/3}p^{1/3}$  and then use Proposition 1 on  $H'$ . It turns out that the choices of  $h$  in [1] were not valid because there is a factor  $g(x)$  in the calculation of  $\gamma(p, w, h)$  which was miscoded and this factor can be negative when  $h$  is large with respect to  $p$ . But smaller  $h$ ’s would avoid this problem and they come at no penalty because the constraint  $(h/2)^{2/3}p^{1/3} \geq H$  is irrelevant. We make the following changes to correct the proof and, in the process, get an improvement:

1. We use Brauer’s inequality  $H < \sqrt{2p} + 2$  for  $p \leq 10^{6.1}$  as opposed to  $p \leq 3 \times 10^6$  from [1]. In this narrower range it implies  $H < 3.38p^{1/4} \log p$ .
2. For  $p \in [10^{6.1}, 10^7]$  we choose  $w = 4$  and  $h = 9$ . This choice of  $w, h$  satisfies all the constraints and  $\gamma(p, w, h) \leq 3.38$ .

3. For  $p \in [10^7, 10^{10}]$  we choose  $w = 5$  and  $h = 17$ .
4. For  $p \in [10^{10}, 10^{13}]$  we choose  $w = 6$  and  $h = 28$ .

We also have mistakes in Remark 2 and Remark 3 of [1]. In Remark 2 we try to prove that Norton's claims are correct, namely that for  $p > e^{15}$ , it is true that  $H(p) < 2.5p^{1/4} \log p$ . The proof once again makes the mistake of taking an  $h$  that does not satisfy the constraints. What we can prove is that  $H(p) < 3p^{1/4} \log p$  for  $p > e^{15}$ . To correct the proof we make the following changes:

1. Let  $w = 4, h = 11$ . Then in the range  $p \in [e^{15}, 10^7]$  we have  $\gamma(p, w, h) < 3$ .
2. Let  $w = 5, h = 16$ . Then in the range  $p \in [10^7, 10^9]$  we have  $\gamma(p, w, h) < 3$ .
3. Let  $w = 6, h = 28$ . Then in the range  $p \in [10^9, 10^{13}]$  we have  $\gamma(p, w, h) < 3$ .

For Remark 3, we should change the bound of  $3p^{1/4} \log p$  to  $3.1p^{1/4} \log p$  for the case of the maximum number of consecutive non-residues for which  $\chi$  remains constant. The proof requires the following changes:

1. We use Hudson's inequality:  $H < p^{1/2} + 2^{2/3}p^{1/3} + 2^{1/3}p^{1/6} + 1$  for  $p \leq 10^{6.4}$  as opposed to  $p \leq 2 \cdot 10^6$ . With this change we get that  $H(p) < 3.1p^{1/4} \log p$  for  $p < 10^{6.4}$ .
2. For  $p \in [10^{6.4}, 10^7]$  we choose  $w = 5$  and  $h = 10$ . Then  $\gamma(p, w, h) \leq 3.1$ .
3. For  $p \in [10^7, 10^9]$  we choose  $w = 5$  and  $h = 16$ . Then  $\gamma(p, w, h) \leq 3.1$ .
4. For  $p \in [10^9, 10^{13}]$  we choose  $w = 6$  and  $h = 36$ .

## Bibliography

1. **E. Treviño**, *On the maximum number of consecutive integers on which a character is constant*, Mosc. J. Comb. Number Theory **2** (1) (2012), 56–72.

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